

Theory of strongly correlated quantum matter (SCQM)

# Multi-Method, Multi-Messenger Approaches to Models of Strong Correlations

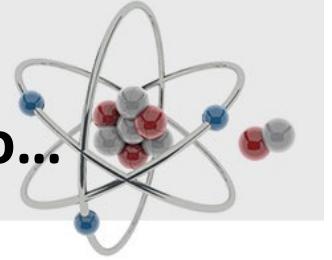
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Thomas Schäfer

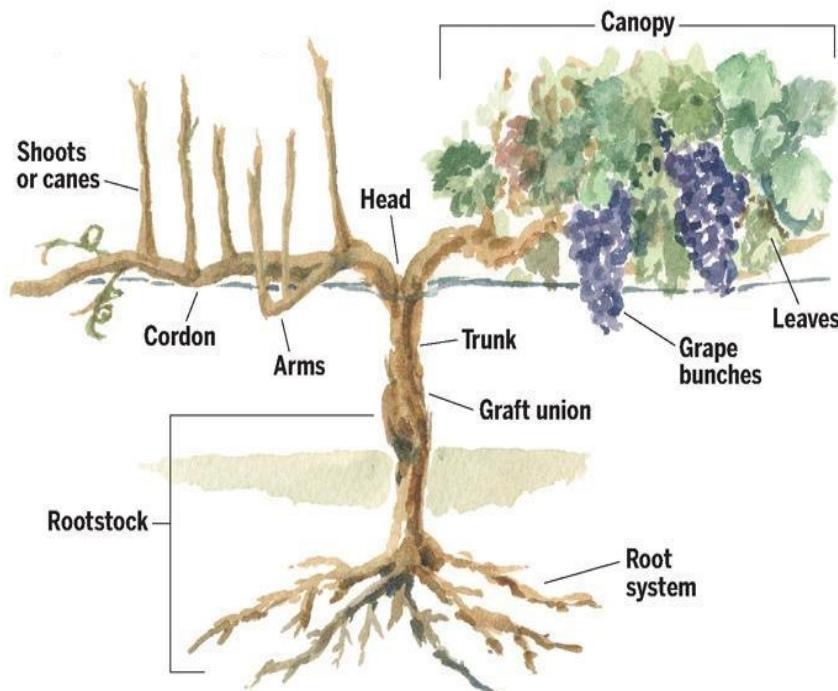
*Head of Max Planck Research Group “Theory of strongly correlated quantum matter” (SCQM)  
MPI for Solid State Research (MPI-FKF), Stuttgart*

Strange metals: from the Hubbard model to AdS/CFT, Institute of Physics Belgrade (Online)  
23<sup>rd</sup> May 2022

# From roots to stems to harvest: the application of multi-method, multi-messenger studies to...



...real materials



Magnetic correlations in infinite-layer nickelates: An experimental and theoretical multimethod study

R. A. Ortiz<sup>1,\*</sup>, P. Puphal,<sup>1,\*</sup> M. Klett,<sup>1</sup> F. Hotz,<sup>2</sup> R. K. Kremer,<sup>1</sup> H. Trepka<sup>1</sup>, M. Hemmida,<sup>3</sup> H.-A. Krug von Nidda,<sup>3</sup> M. Isobe,<sup>1</sup> R. Khasanov<sup>1,2</sup>, H. Luetkens,<sup>2</sup> P. Hansmann<sup>1,4</sup>, B. Keimer,<sup>1</sup> T. Schäfer<sup>1,1†</sup>, and M. Hepting<sup>1,1‡</sup>

*Phys. Rev. Research 4, 023093 (2022)*

...uncharted (model) territory

Mott Insulating States with Competing Orders in the Triangular Lattice Hubbard Model

Alexander Wietek<sup>1,\*</sup>, Riccardo Rossi<sup>1,2</sup>, Fedor Šimkovic IV<sup>3,4</sup>, Marcel Klett,<sup>5</sup> Philipp Hansmann<sup>6</sup>, Michel Ferrero<sup>3,4</sup>, E. Miles Stoudenmire<sup>1</sup>, Thomas Schäfer<sup>5</sup>, and Antoine Georges<sup>4,1,3,7</sup>

*Physical Review X 11, 041013 (2021)*

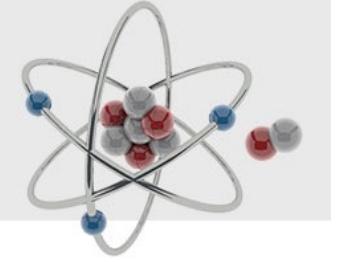
...a simple (?) test case

Tracking the Footprints of Spin Fluctuations: A MultiMethod, MultiMessenger Study of the Two-Dimensional Hubbard Model

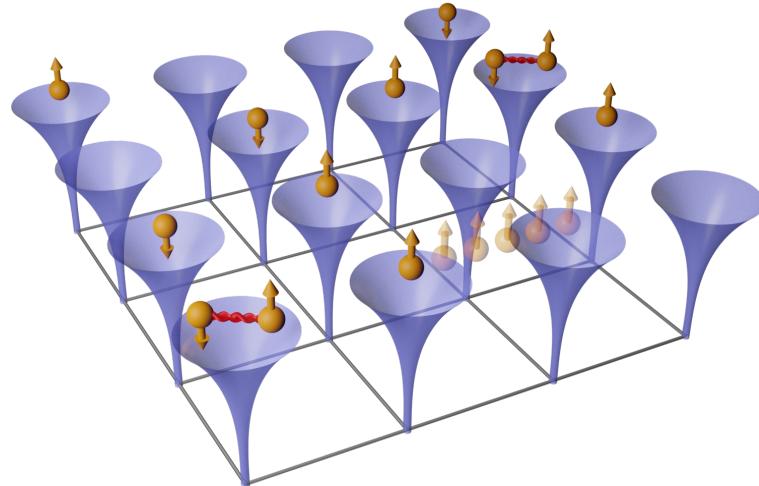
Thomas Schäfer<sup>1,2,3,\*</sup>, Nils Wentzell<sup>4</sup>, Fedor Šimkovic IV<sup>1,2</sup>, Yuan-Yao He<sup>4,5</sup>, Cornelia Hille<sup>6</sup>, Marcel Klett,<sup>6,3</sup> Christian J. Eckhardt<sup>7,8</sup>, Behnam Arzhang,<sup>9</sup> Viktor Harkov<sup>10,11</sup>, François-Marie Le Régent<sup>12</sup>, Alfred Kirsch,<sup>2</sup> Yan Wang,<sup>12</sup> Aram J. Kim<sup>13</sup>, Evgeny Kozik<sup>13</sup>, Evgeny A. Stepanov<sup>10</sup>, Anna Kauch<sup>13</sup>, Sabine Andergassen<sup>14</sup>, Philipp Hansmann<sup>14,15</sup>, Daniel Rohe<sup>16</sup>, Yuri M. Vilk,<sup>12</sup> James P. F. LeBlanc<sup>19</sup>, Shiwei Zhang<sup>14,5</sup>, A.-M. S. Tremblay<sup>12</sup>, Michel Ferrero<sup>1,2</sup>, Olivier Parcollet<sup>4,17</sup>, and Antoine Georges<sup>1,2,4,18</sup>

*Physical Review X 11, 011058 (2021)*

# Strongly correlated systems: fascinating physics - and a simple (?) modellization



## Hubbard model



$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

**-t** : hopping

**U**: local Coulomb interaction



*Annual Review of Condensed Matter Physics*  
The Hubbard Model:  
A Computational  
Perspective

Mingpu Qin,<sup>1</sup> Thomas Schäfer,<sup>2</sup> Sabine Andergassen,<sup>3</sup>  
Philippe Corboz,<sup>4</sup> and Emanuel Gull<sup>5</sup>

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<sup>3</sup>Institut für Theoretische Physik and Center for Quantum Science, Universität Tübingen, Germany

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Annu. Rev. Condens. Matter Phys. 2022. 13:275–302  
The *Annual Review of Condensed Matter Physics* is online at [commatphys.annualreviews.org](http://commatphys.annualreviews.org)  
<https://doi.org/10.1146/annurev-commatphys-090921-031948>

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### Keywords

quantum many-body theory, model Hamiltonians, strongly correlated electron systems

### Abstract

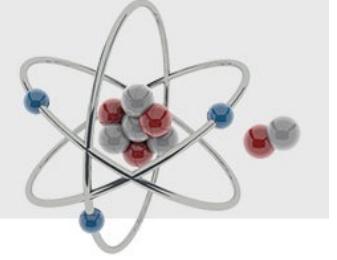
The Hubbard model is the simplest model of interacting fermions on a lattice and is of similar importance to correlated electron physics as the Ising model is to statistical mechanics or the fruit fly to biomedical science. Despite its simplicity, the model exhibits an incredible wealth of phases, phase transitions, and exotic correlation phenomena. Although analytical methods have provided a qualitative description of the model in certain limits, numerical tools have shown impressive progress in achieving quantitative accurate results over the past several years. This article gives an introduction to the model, motivates common questions, and illustrates the progress that has been achieved over recent years in revealing various aspects of the correlation physics of the model.

275

J. Hubbard, Proc. Royal Soc. A, **276**, 238–257 (1963)

M. Qin, TS, et al., „The Hubbard model – a computational perspective“, *Annual Review of Condensed Matter Physics* **13** (2022)

# Several ways to attack the Hubbard model



## Numerically exact techniques

- Lattice quantum Monte Carlo (LQMC)
- Diagrammatic Monte Carlo (DMC)

Let us put these methods  
to the test...

## Mean field theory

(MFT)

Dynamic mean field theory (DMFT)

$$H = -t \sum c_{i\sigma}^\dagger c_{j\sigma} + U \sum n_{i\uparrow} n_{i\downarrow}$$

PHYSICAL REVIEW X 11, 011058 (2021)

## Extensions

### Cluster extensions

- Dynamical cluster theory (DCT)
- cell dynamical cluster theory (CDCT)

### Diagrammatic extensions (larger scales)

- **Dynamical vertex approximation** (DΓA)
- **Dual fermion** (DF)
- **Dual boson** (DB)
- **Triply irreducible local expansion** (TRILEX)

## Techniques

### Coherent approach

### Quantum group

## Tracking the Footprints of Spin Fluctuations: A MultiMethod, MultiMessenger Study of the Two-Dimensional Hubbard Model

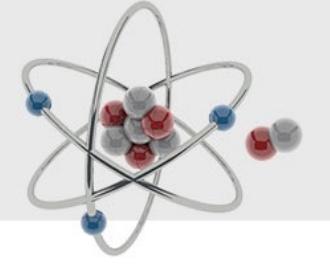
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(PRX)

- **Parquet approximation** (PA)

- [tensor networks: DMRG, METTS, etc.]

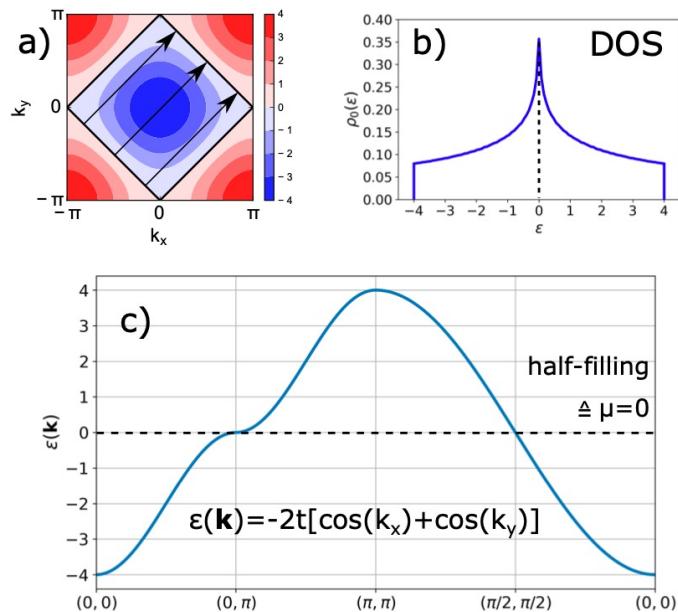
# Putting all these methods to the test



PHYSICAL REVIEW X 11, 011058 (2021)

## Tracking the Footprints of Spin Fluctuations: A MultiMethod, MultiMessenger Study of the Two-Dimensional Hubbard Model

Thomas Schäfer<sup>1,2,3,\*</sup>, Nils Wentzell<sup>4</sup>, Fedor Šimkovic IV,<sup>1,2</sup> Yuan-Yao He,<sup>4,5</sup> Cornelia Hille<sup>6</sup>, Marcel Klett,<sup>6,3</sup> Christian J. Eckhardt<sup>7,8</sup>, Behnam Arzhang,<sup>9</sup> Viktor Harkov<sup>10,11</sup>, François-Marie Le Régent<sup>2</sup>, Alfred Kirsch,<sup>2</sup> Yan Wang,<sup>12</sup> Aaram J. Kim<sup>13</sup>, Evgeny Kozik<sup>13</sup>, Evgeny A. Stepanov<sup>10</sup>, Anna Kauch<sup>7</sup>, Sabine Andergassen<sup>6</sup>, Philipp Hansmann<sup>14,15</sup>, Daniel Rohe<sup>16</sup>, Yuri M. Vilk,<sup>12</sup> James P. F. LeBlanc<sup>9</sup>, Shiwei Zhang<sup>4,5</sup>, A.-M. S. Tremblay<sup>12</sup>, Michel Ferrero<sup>1,2</sup>, Olivier Parcollet<sup>17</sup>, and Antoine Georges<sup>1,2,4,18</sup>



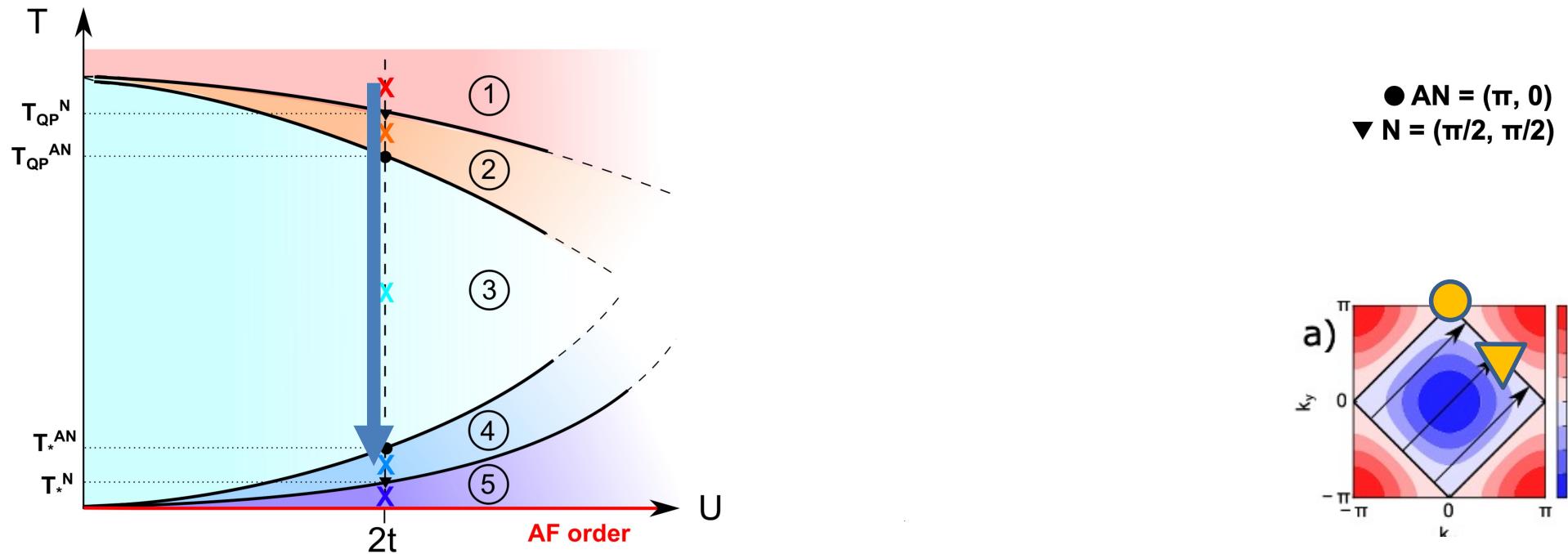
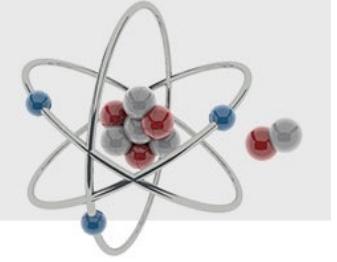
### Test parameters

- 2d unfrustrated square lattice
- Half-filled
- $U=2t$

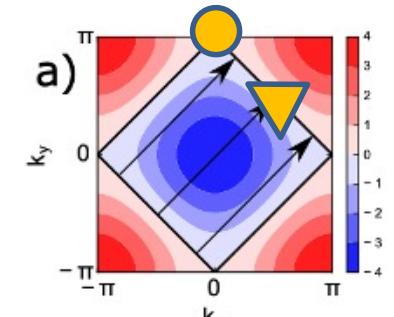
### Observables (as function of T)

- Coherence temperature  $T_{QP}$
- "Pseudogap" temperature  $T_*$
- Double occupancy D
- Magnetic susceptibility  $\chi$
- Magnetic correlation length  $\xi$

# A tale of energy scales

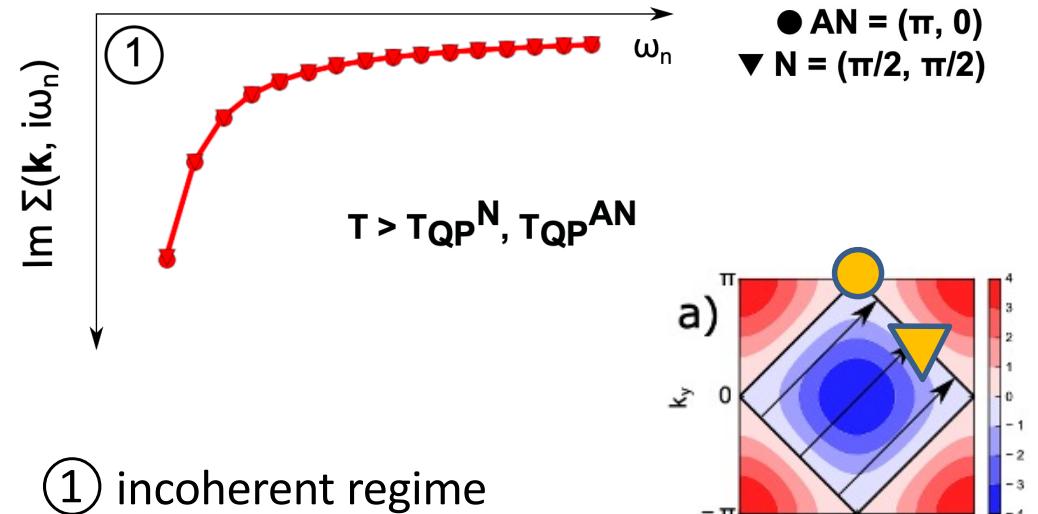
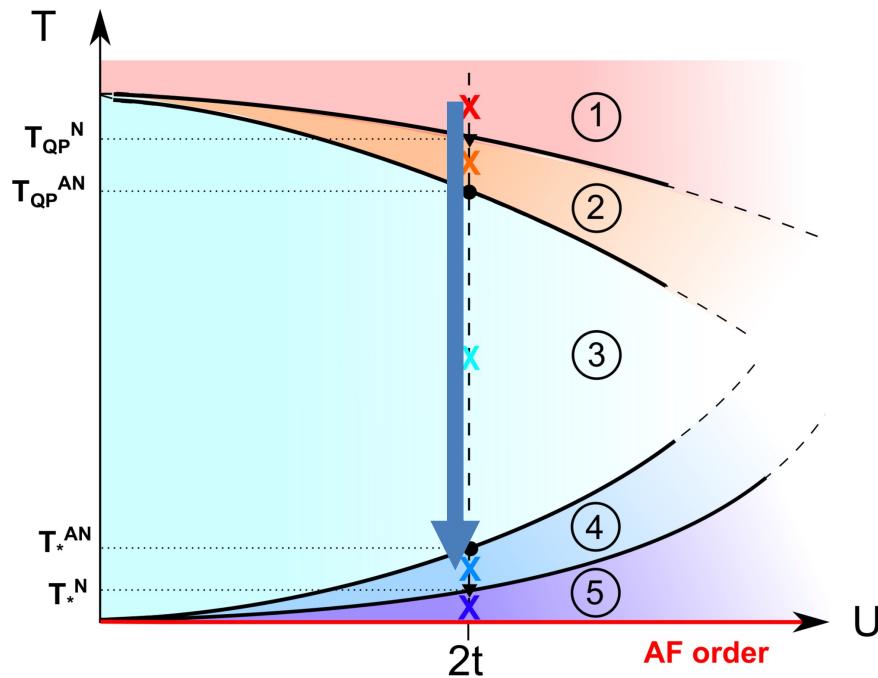
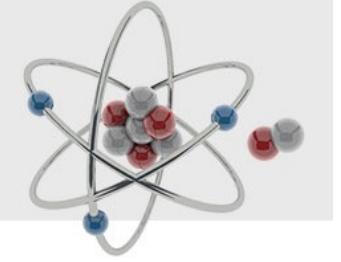


AF order only at  $T=0$  (Mermin-Wagner theorem)

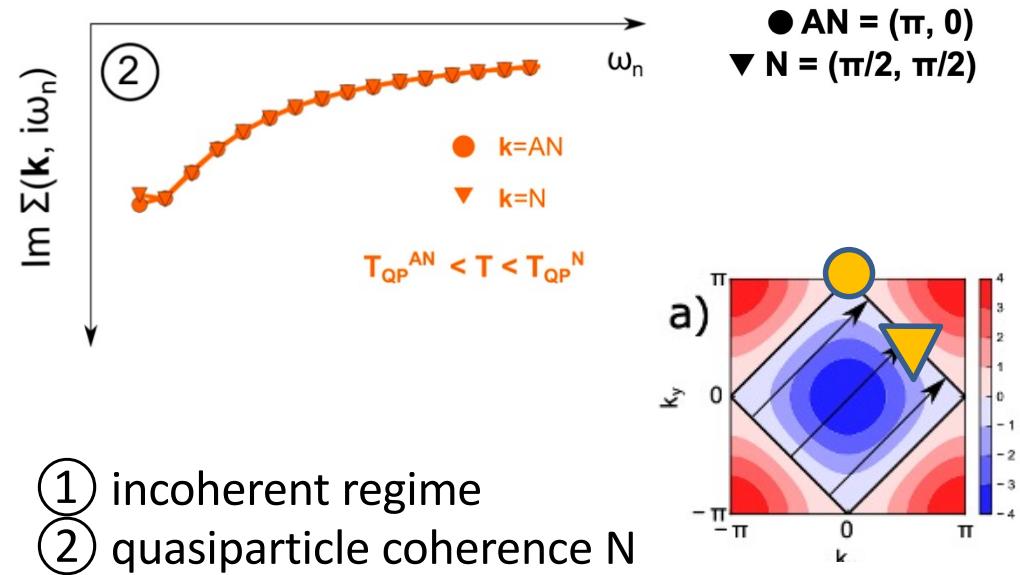
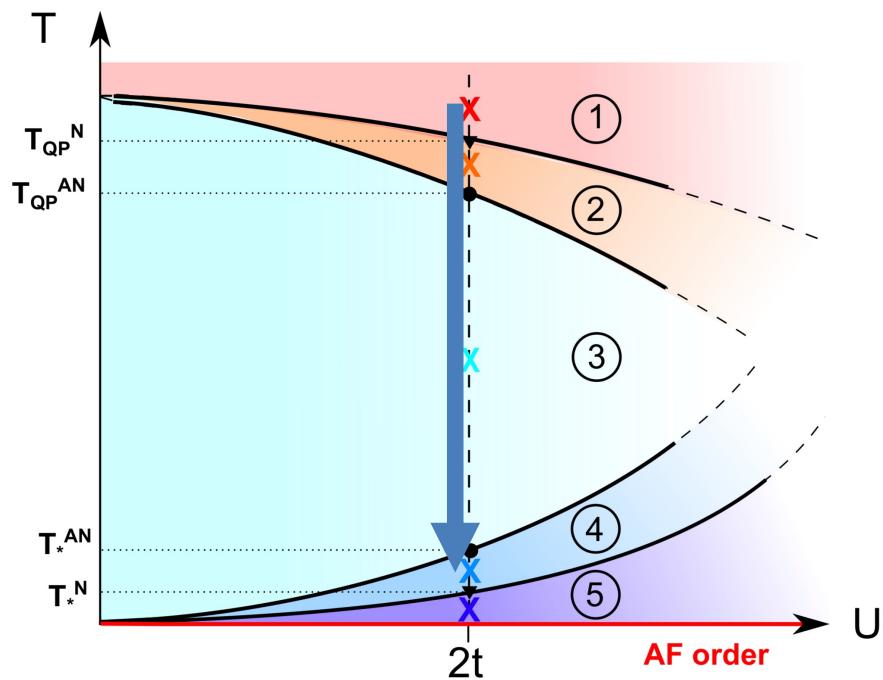
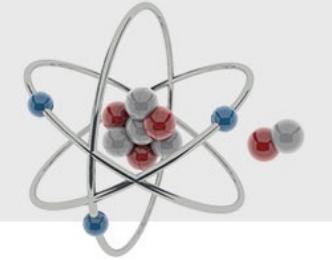


- T. Schäfer et al., Phys. Rev. X **11**, 011058 (2021)*  
*T. Schäfer et al., Phys. Rev. B **91**, 125109 (2015)*  
*F. Šimkovic et al., Phys. Rev. Lett. **124**, 017003 (2020), A. J. Kim, et al., Phys. Rev. Lett. **124**, 117602 (2020)*

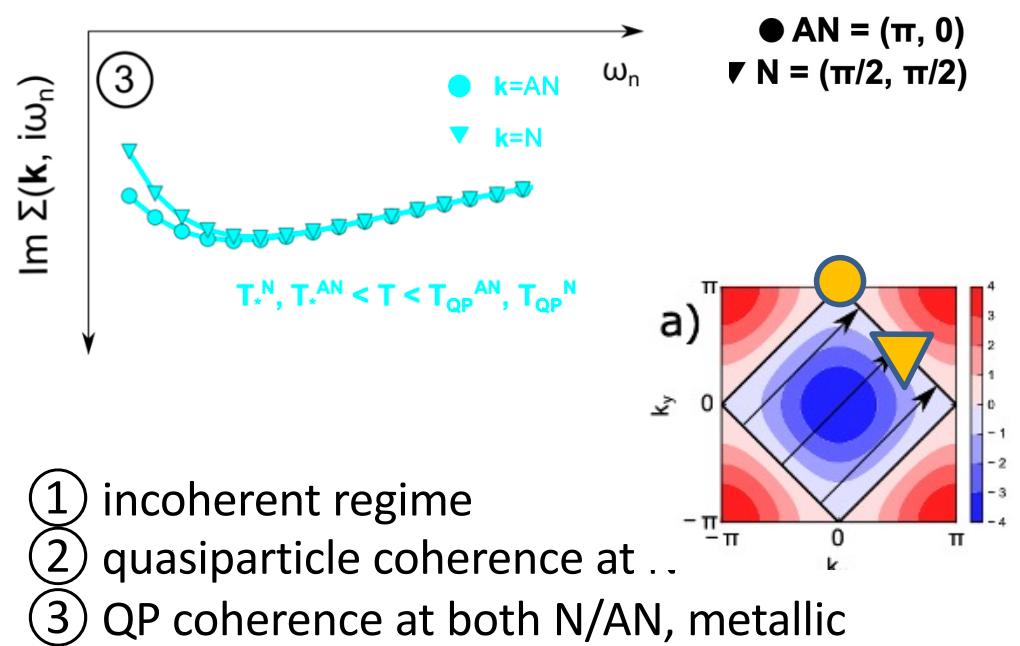
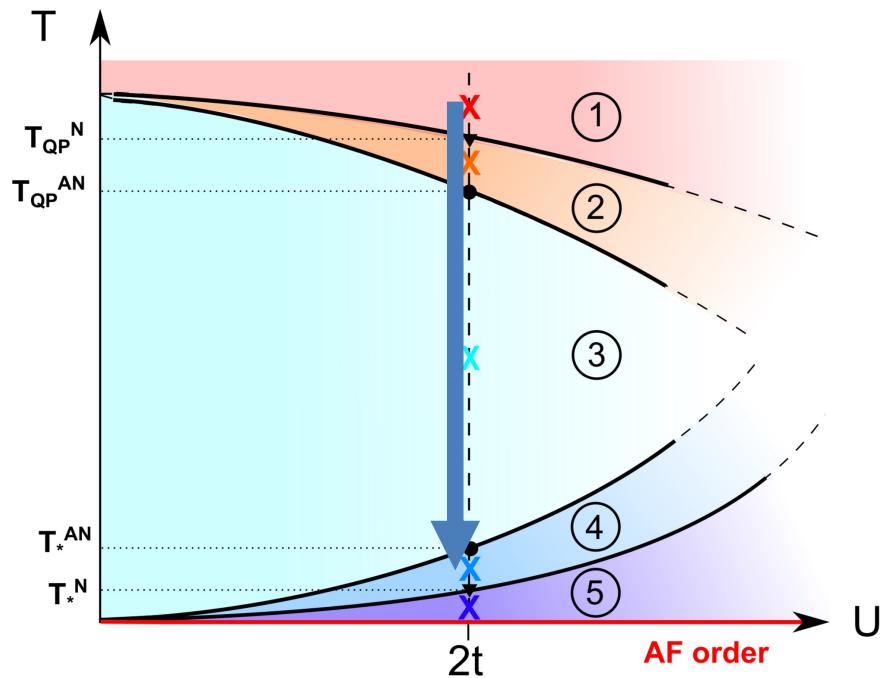
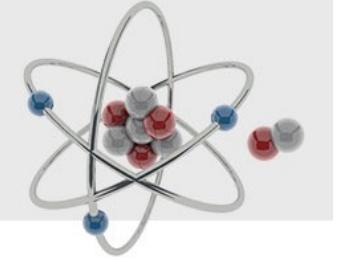
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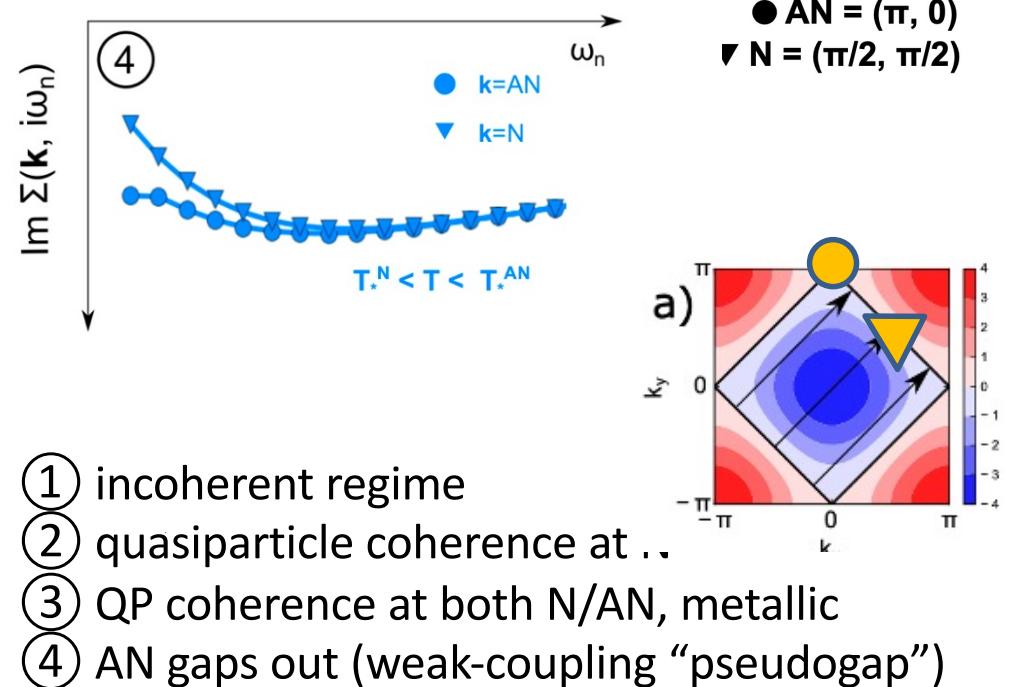
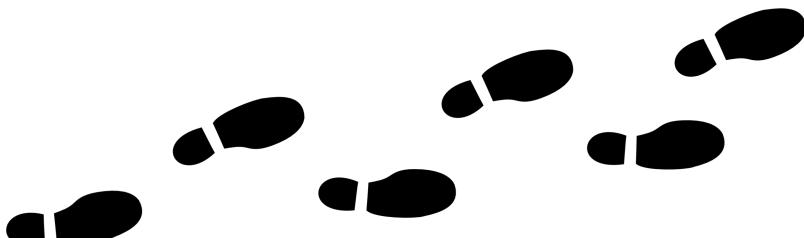
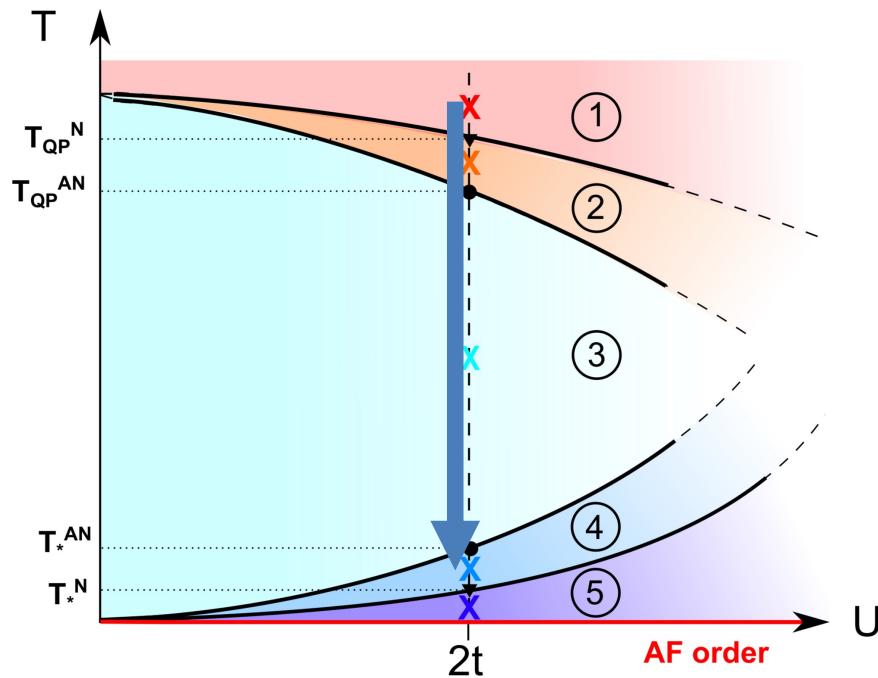
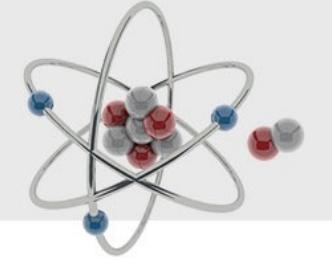
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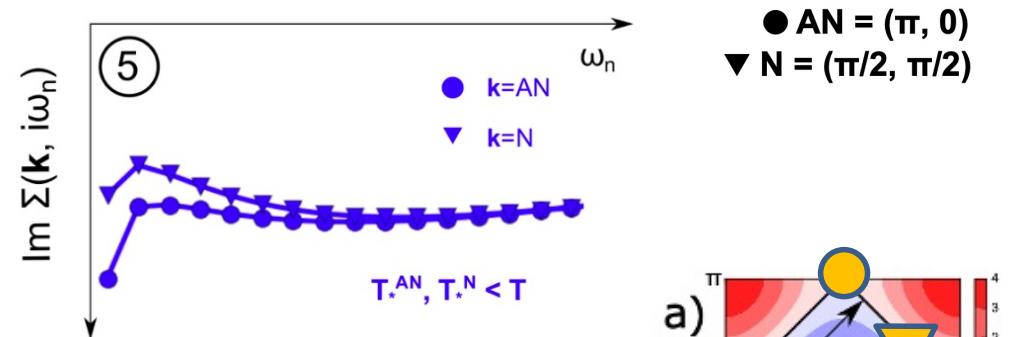
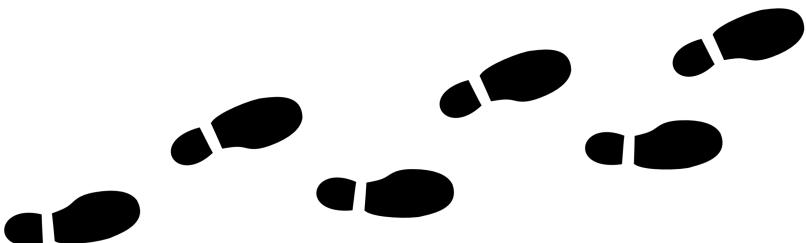
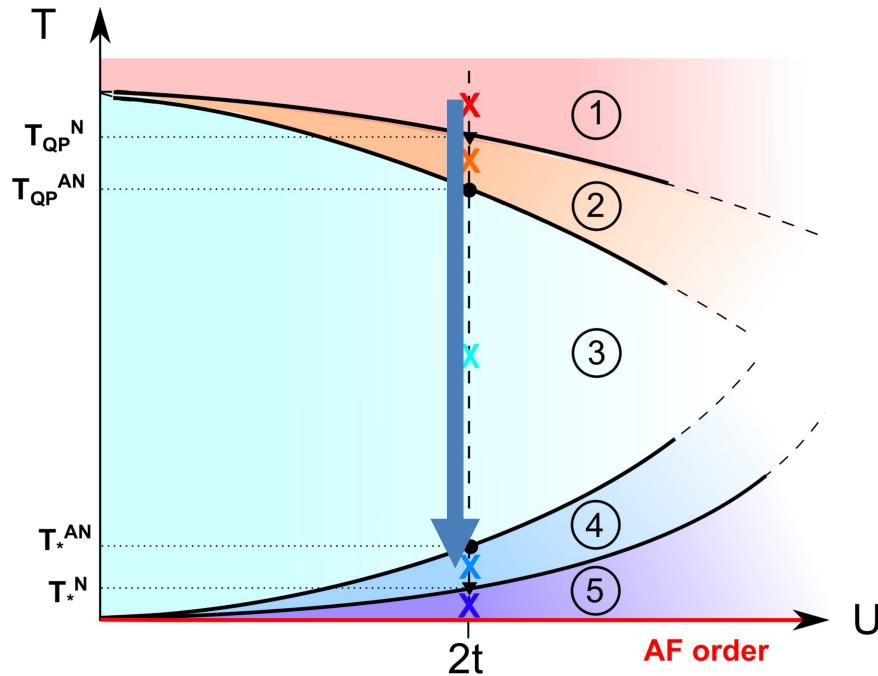
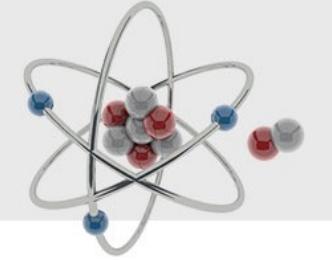
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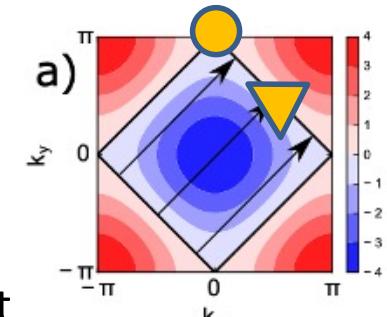
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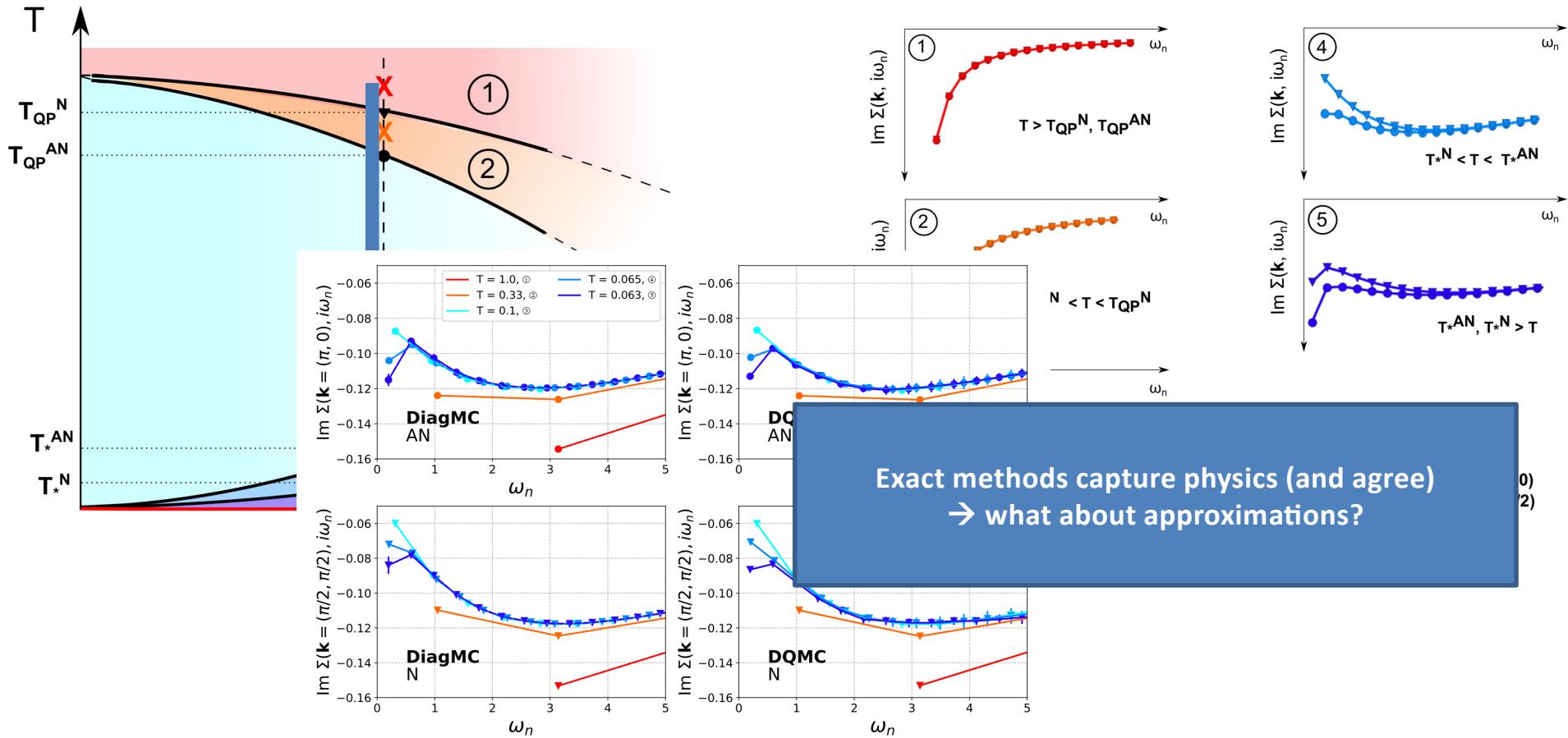
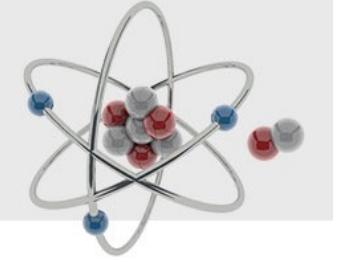
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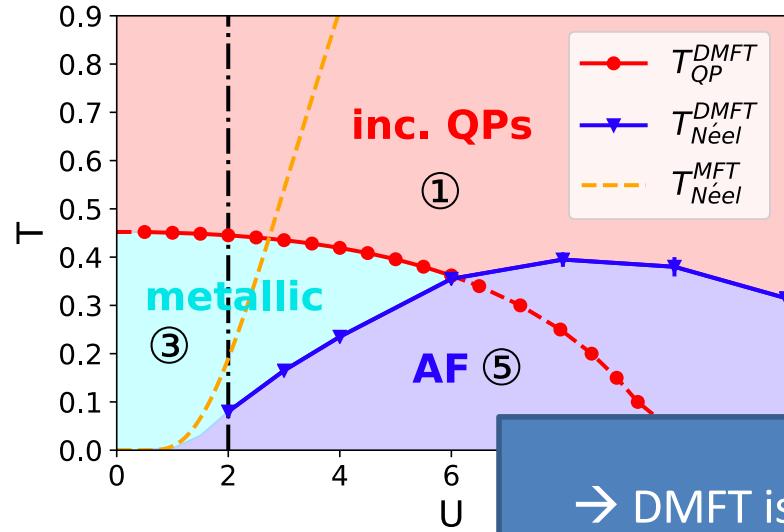
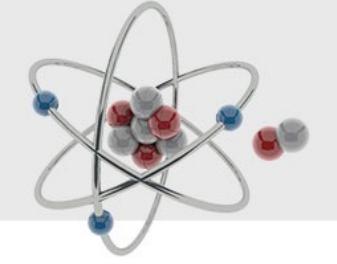
- ① incoherent regime
- ② quasiparticle coherence at ...
- ③ QP coherence at both N/AN, metallic
- ④ AN gaps out (weak-coupling “pseudogap”)
- ⑤ N gaps out, insulating regime



# A tale of energy scales

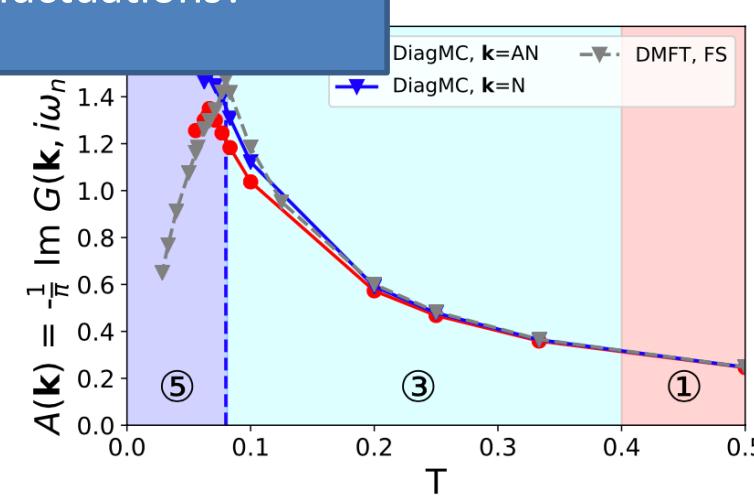
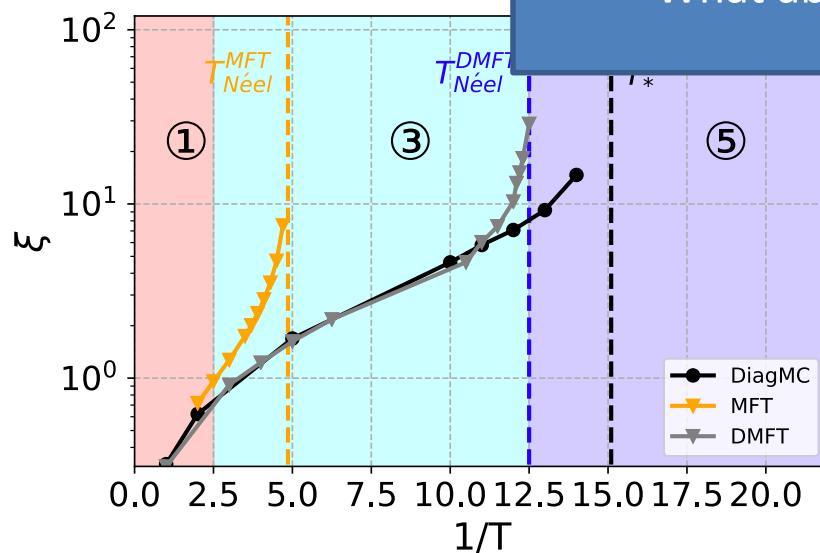


# A tale of energy scales: the dynamical MF starting point

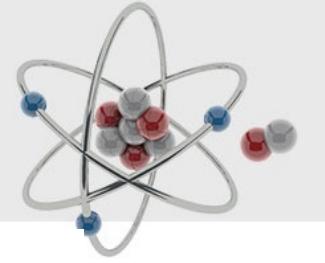


- DMFT: does not satisfy Mermin-Wagner ( $\rightarrow$  finite  $T_N$ )
- Captures the crossover from metallic to insulating via a **phase transition** (finite magnetization)

→ DMFT is a good starting point  
What about fluctuations?



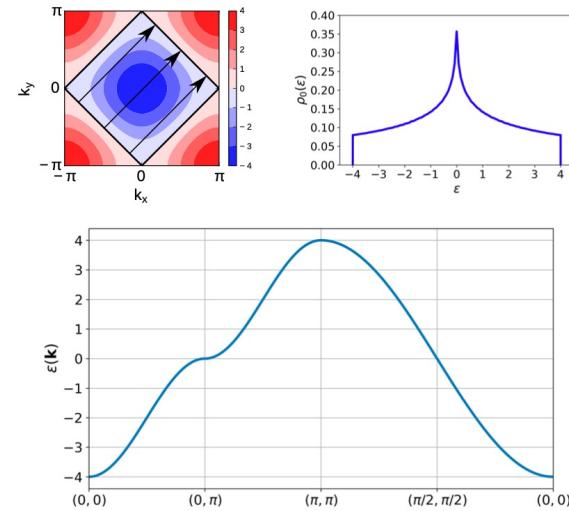
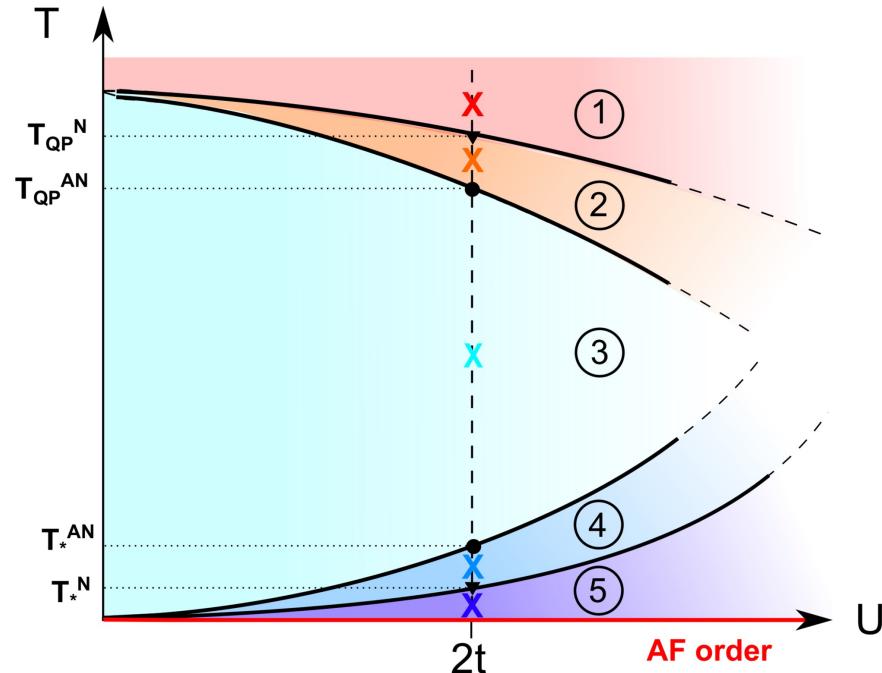
# Measuring fluctuations: two-particle quantities



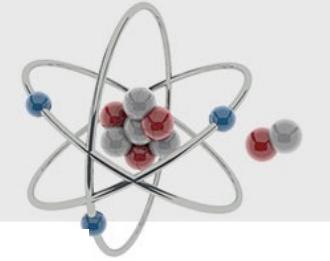
## Static antiferromagnetic susceptibility

$$\chi_{\text{sp}}(\mathbf{q}, i\Omega_n) = \int_0^\beta d\tau \sum_{\mathbf{r}} e^{i\tau\Omega_n} e^{-i\mathbf{qr}} \langle S_z(\mathbf{r}, \tau) S_z(0, 0) \rangle$$

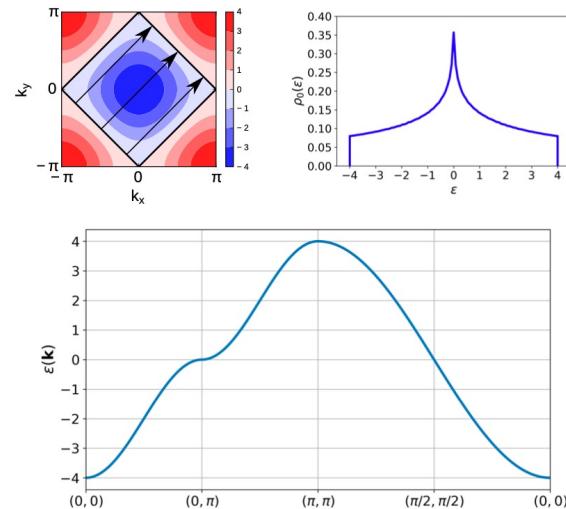
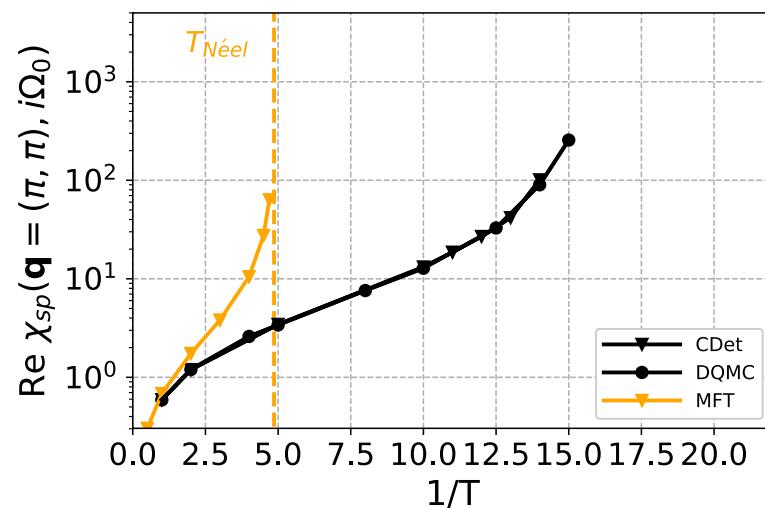
$$S_z(\mathbf{r}, \tau) = n_\uparrow(\mathbf{r}, \tau) - n_\downarrow(\mathbf{r}, \tau)$$



# Measuring fluctuations: two-particle quantities

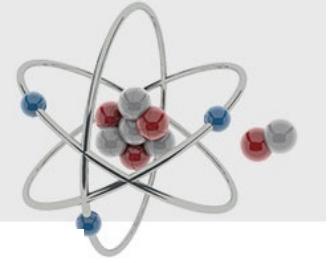


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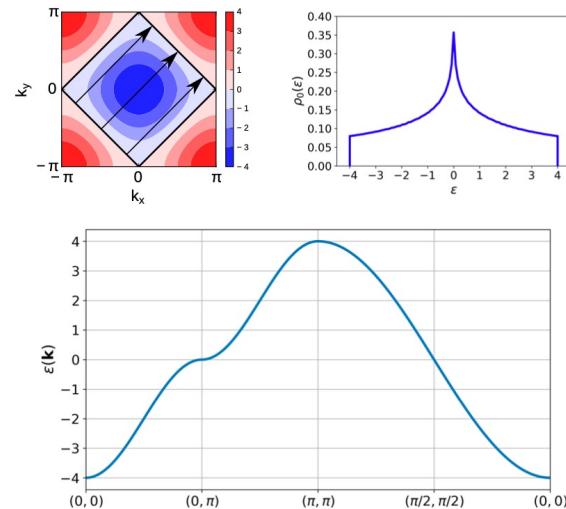
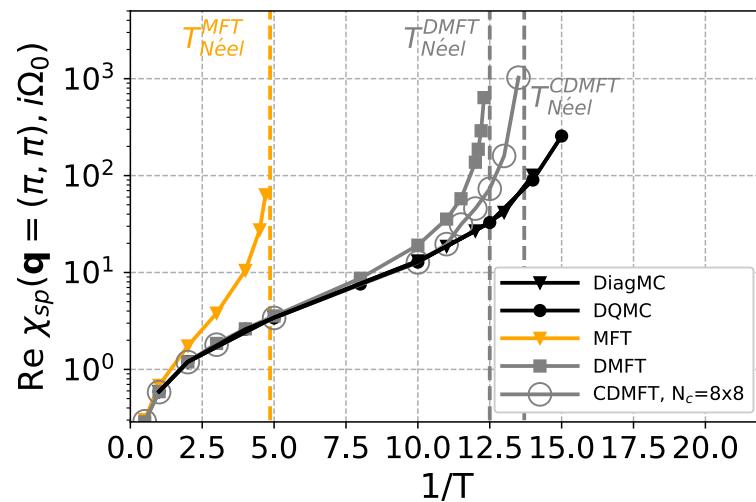


	DiagMC	DQMC
T_N	✓	0
$\chi(T=0.1)$	13.2	13.2

# Measuring fluctuations: two-particle quantities

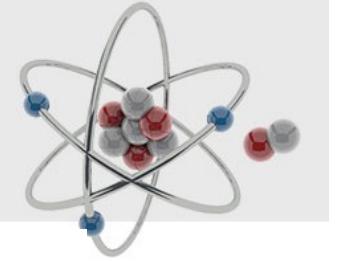


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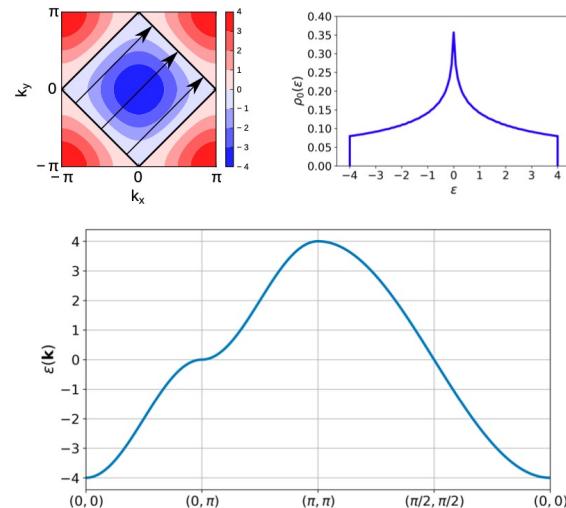
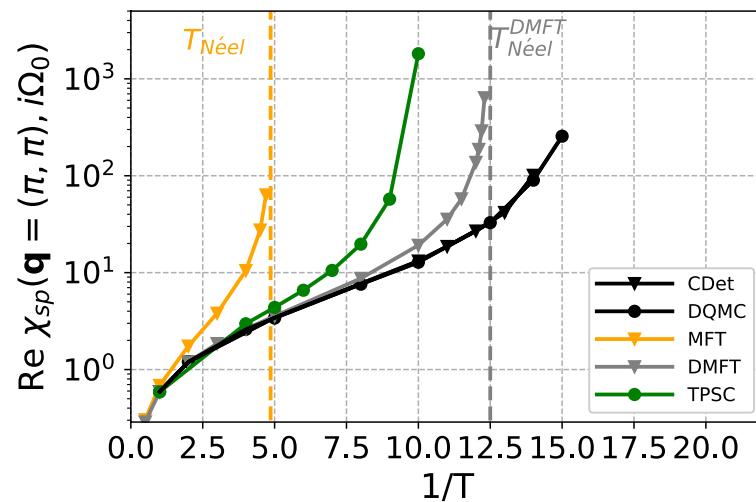


	DiagMC	DQMC	DMFT
$T_N$	✓ 0	✓ 0	✗ 0.08
$\chi(T=0.1)$	13.2	13.2	19.2

# Measuring fluctuations: two-particle quantities

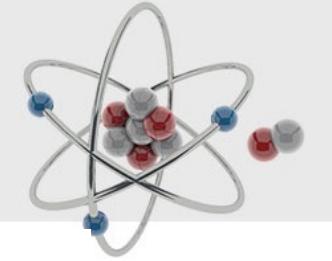


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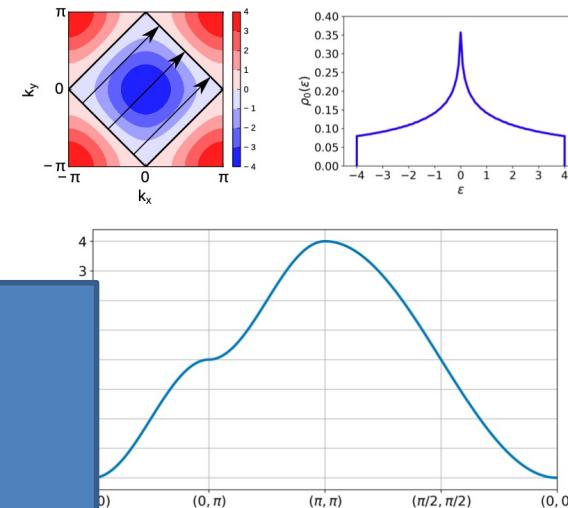
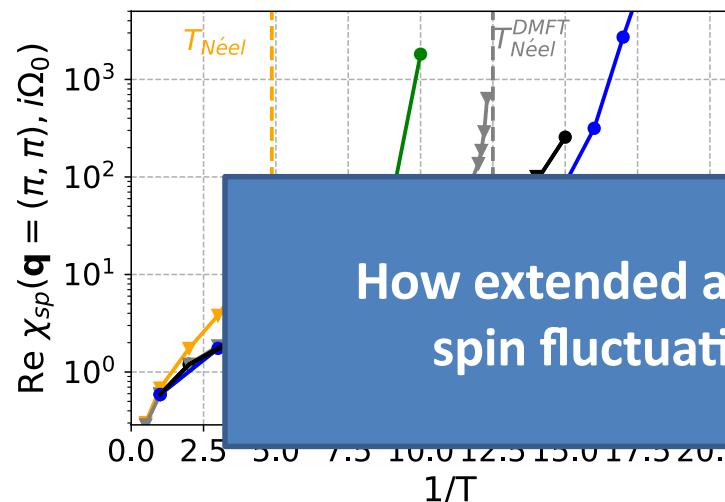


	DiagMC	DQMC	DMFT	TPSC
$T_N$	✓ 0	✓ 0	✗ 0.08	✓ 0
$\chi(T=0.1)$	13.2	13.2	19.2	1816.5

# Measuring fluctuations: two-particle quantities

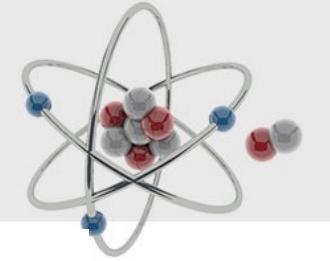


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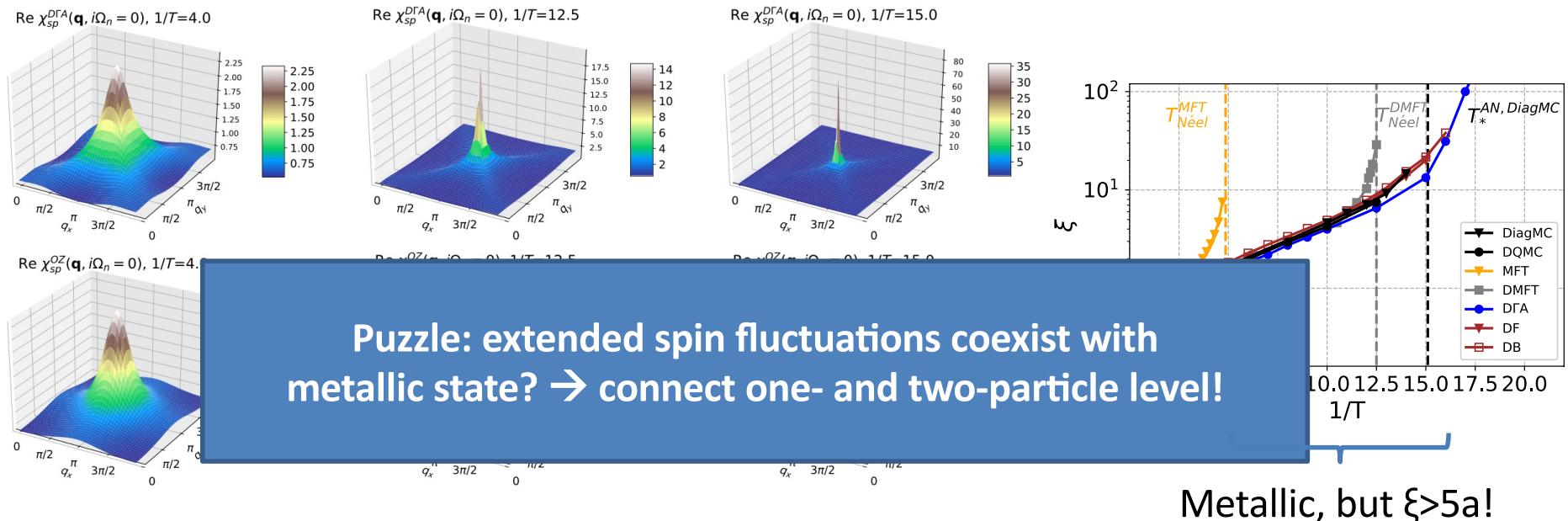
	DiagMC	DQMC	DMFT	TPSC	DΓA
$T_N$	✓ 0	✓ 0	✗ 0.08	✓ 0	✓ 0
$\chi(T=0.1)$	13.2	13.2	19.2	1816.5	15.3

# Measuring fluctuations: two-particle quantities



## Magnetic correlation length

via Ornstein-Zernike fits  $\chi(\mathbf{q}, i\Omega_n) = \frac{A}{(\mathbf{q} - \mathbf{Q})^2 + \xi^{-2} + \frac{|\Omega_n|}{\gamma}}$



Magnetic correlation length exponentially growing!

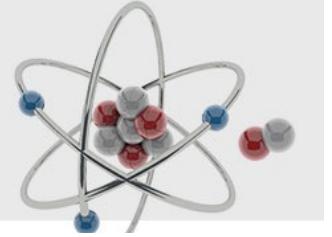
Condition for **pseudogap** at weak coupling (Vilk criterion):

$$\frac{\nu_F}{\pi T} \ll \xi$$

→ Footprints of spin fluctuations in all observables  
(on the one- and two-particle level)



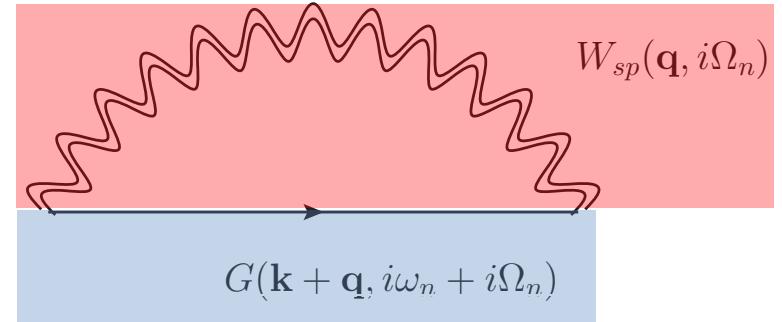
# The (fairly “strange”) **metallic** regime I: Handshake with analytical spin fluctuation theory



$$\Sigma_{SF}(\mathbf{k}, i\omega_n) = \text{Non-interacting } G_0$$

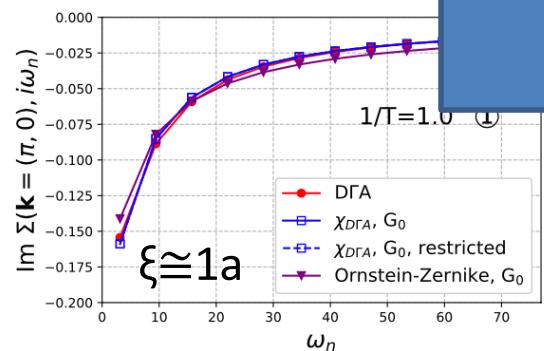
$$g^2 T \int \frac{d^2 q}{(2\pi)^2} \sum_{\Omega_n} G(\mathbf{k} + \mathbf{q}, i\omega_n + i\Omega_n) \chi(\mathbf{q}, i\Omega_n)$$

coupling of fermion to spin modes  $\frac{3}{8} U^2$



Handshake with nume

How can extended spin fluctuations coexist with metallic behavior?

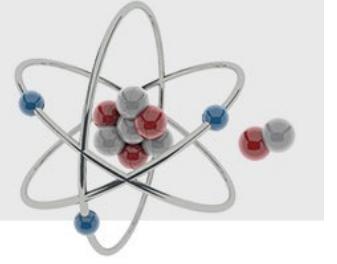


Three choices for susceptibility

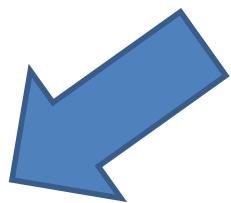
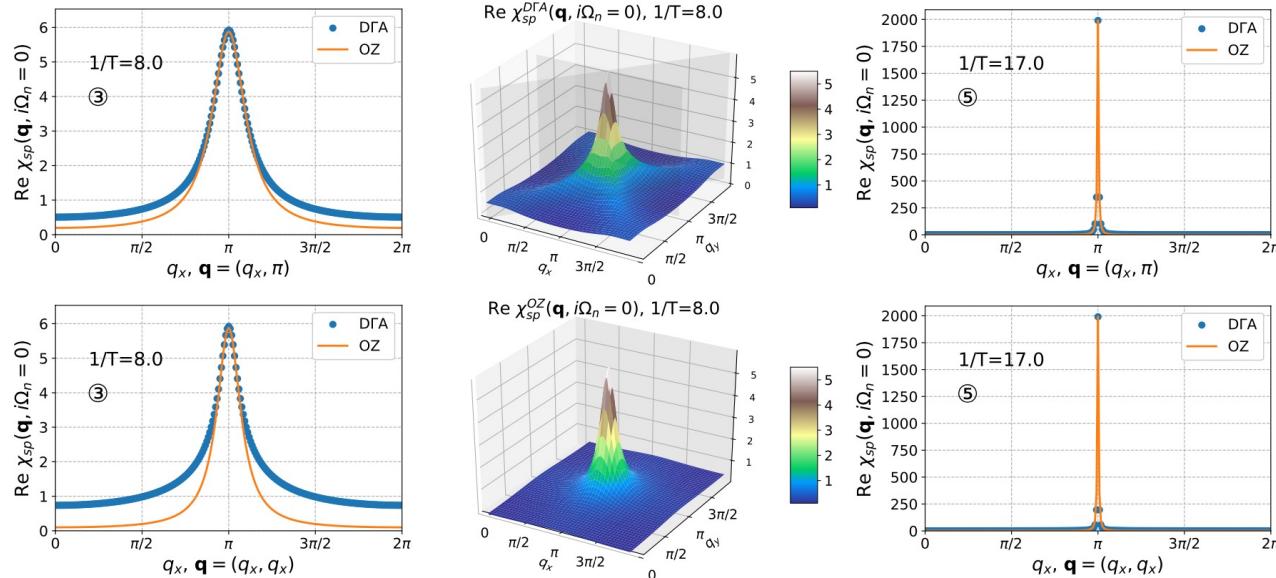
- Full DΓA spin susceptibility
- DΓA spin susceptibility restricted to  $|q_i - Q_i| < 2\xi^{-1}$  around AF peak
- Ornstein-Zernike fit

} Non-metallic self-energy in regime ③?!

# The (fairly “strange”) **metallic** regime I: Handshake with analytical **spin fluctuation theory**



$$\Sigma_{\text{SF}}(\mathbf{k}, i\omega_n) = g^2 T \int \frac{d^2 q}{(2\pi)^2} \sum_{\Omega_n} G(\mathbf{k} + \mathbf{q}, i\omega_n + i\Omega_n) \chi(\mathbf{q}, i\Omega_n)$$



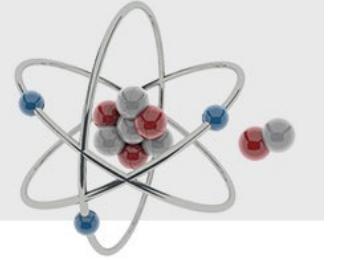
→ Resolves dichotomy of sizeable correlation length and metallic state



Many  $\mathbf{q}$ -vectors contribute → metallic

$\mathbf{q}=(\pi, \pi)$  is dominant → insulating

# The (fairly “strange”) **metallic** regime II: Perfect nesting and non-Fermi-liquid behavior



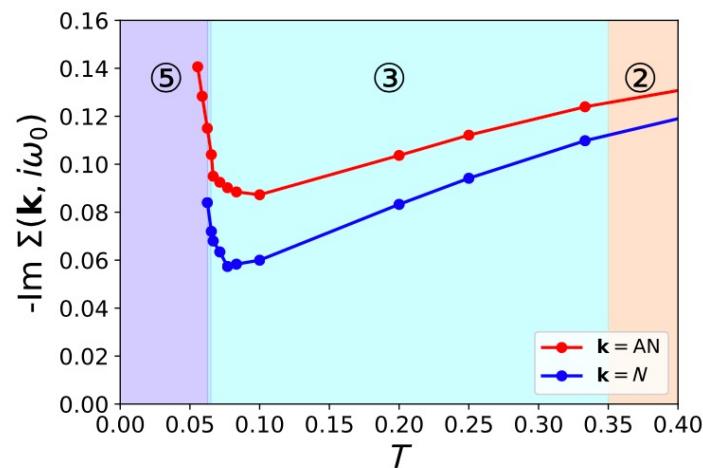
How does perfect nesting influence the **nature of the metal**?

Fermi liquid self-energy on real frequencies:

$$-\text{Im } \Sigma(\mathbf{k}_F, \omega) \propto \underbrace{\omega^2 + (\pi T)^2}_{\dots} + \dots$$

$-\omega_n^2 + (\pi T)^2 \longrightarrow$  vanishes for first Matsubara frequency

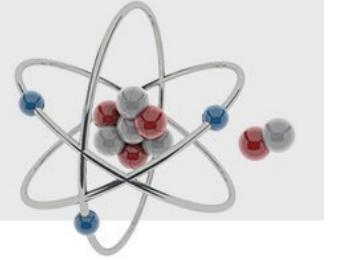
$$\Rightarrow \text{Im } \Sigma(\mathbf{k}_F, \omega_0) \sim \left(1 - \frac{1}{Z}\right) \omega_0 = \left(1 - \frac{1}{Z}\right) \pi T \longrightarrow \text{Linear in } T, \\ \text{“first Matsubara frequency rule”}$$



Data from DiagMC indicates non-Fermi-liquid!  
→ real-frequency data?

- A. Virosztek and J. Ruvalds, *Phys. Rev. B* **42**, 4064 (1990)  
 A. V. Chubukov and D. L. Maslov, *Phys. Rev. B* **86**, 155136 (2012)  
 T. Schäfer et al., *Phys. Rev. X* **11**, 011058 (2021)

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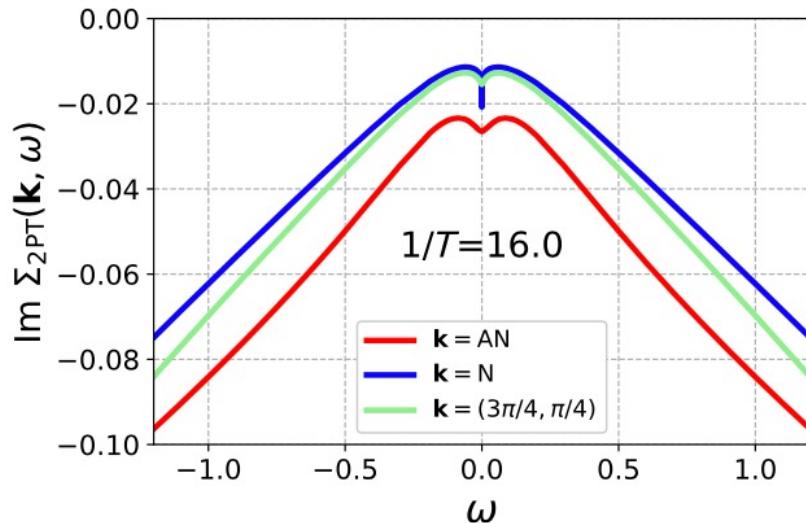
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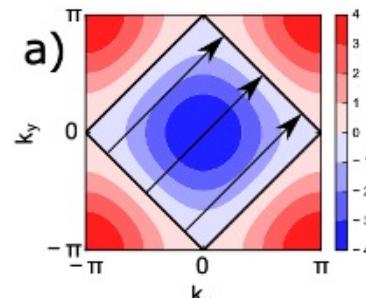
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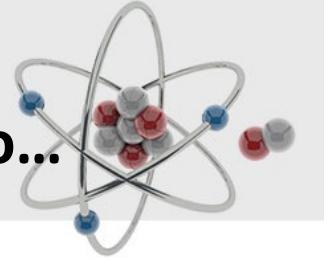


Real frequency data from second Matsubara mode  
→ wrong slope at  $|\omega| < \pi T$

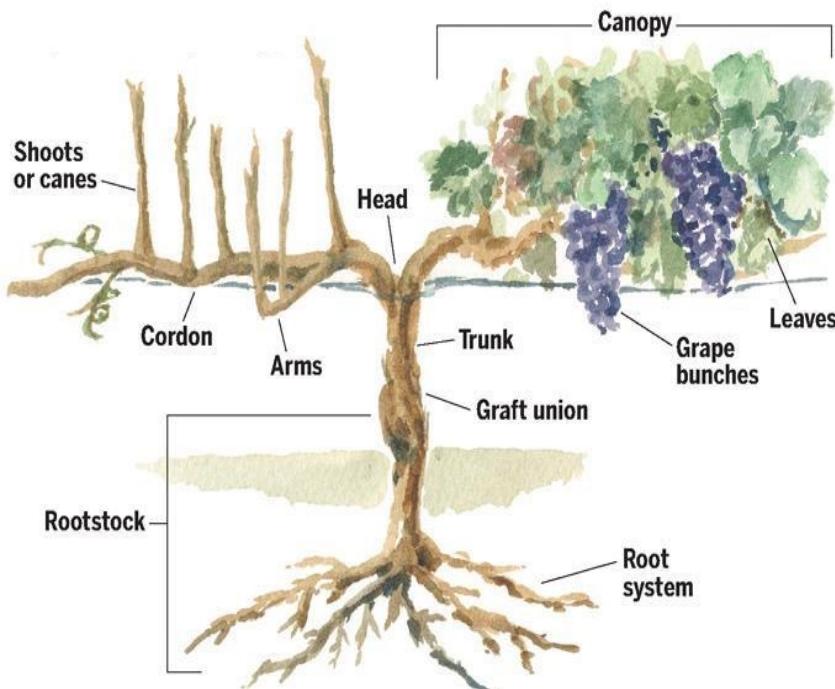
Real frequency data  
highly desired!



# From roots to stems to harvest: the application of multi-method, multi-messenger studies to...



...real materials



Magnetic correlations in infinite-layer nickelates: An experimental and theoretical multimethod study

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*Phys. Rev. Research 4, 023093 (2022)*

...uncharted (model) territory

Mott Insulating States with Competing Orders in the Triangular Lattice Hubbard Model

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*Physical Review X 11, 041013 (2021)*

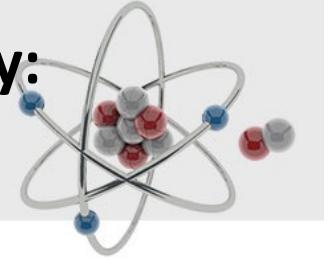
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Tracking the Footprints of Spin Fluctuations: A MultiMethod, MultiMessenger Study of the Two-Dimensional Hubbard Model

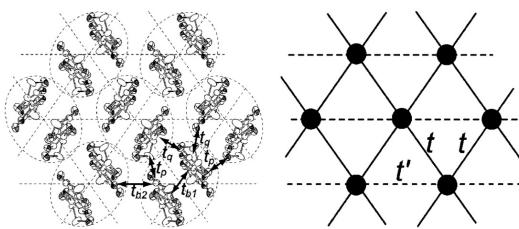
Thomas Schäfer<sup>1,2,3,\*</sup>, Nils Wentzell<sup>4</sup>, Fedor Šimkovic IV<sup>1,2</sup>, Yuan-Yao He<sup>4,5</sup>, Cornelia Hille<sup>6</sup>, Marcel Klett,<sup>6,3</sup> Christian J. Eckhardt<sup>7,8</sup>, Behnam Arzhang,<sup>9</sup> Viktor Harkov<sup>10,11</sup>, François-Marie Le Régent<sup>12</sup>, Alfred Kirsch,<sup>2</sup> Yan Wang,<sup>12</sup> Aram J. Kim<sup>13</sup>, Evgeny Kozik<sup>13</sup>, Evgeny A. Stepanov<sup>10</sup>, Anna Kauch<sup>7</sup>, Sabine Andergassen<sup>6</sup>, Philipp Hansmann<sup>14,15</sup>, Daniel Rohe<sup>16</sup>, Yuri M. Vilk,<sup>12</sup> James P. F. LeBlanc<sup>9</sup>, Shiwei Zhang<sup>4,5</sup>, A.-M. S. Tremblay<sup>12</sup>, Michel Ferrero<sup>1,2</sup>, Olivier Parcollet<sup>4,17</sup>, and Antoine Georges<sup>1,2,4,18</sup>

*Physical Review X 11, 011058 (2021)*

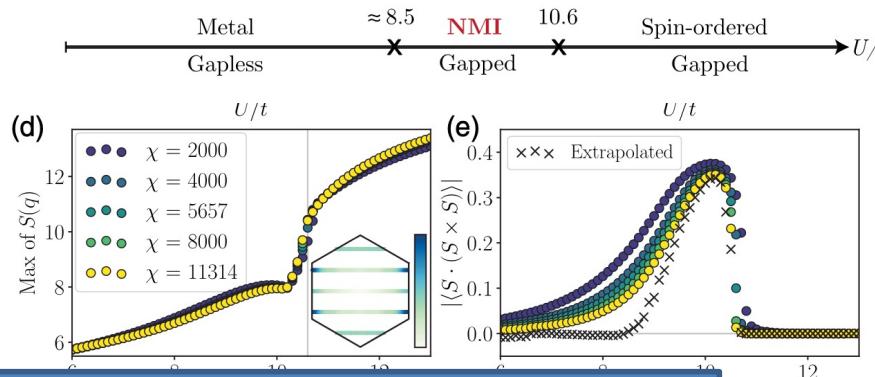
# A multimethod, multimesenger study in uncharted territory: The Hubbard model on the triangular lattice



Potential spin-liquid candidate: organic crystal  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub>

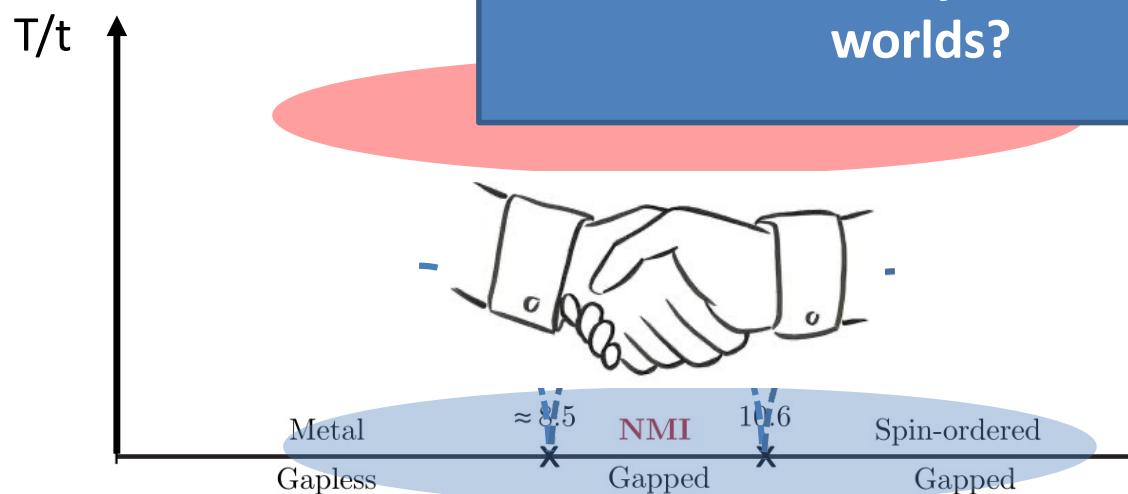


Hubbard model on  
triangular lattice



iDMRG  
calculations

Can we marry these two  
worlds?



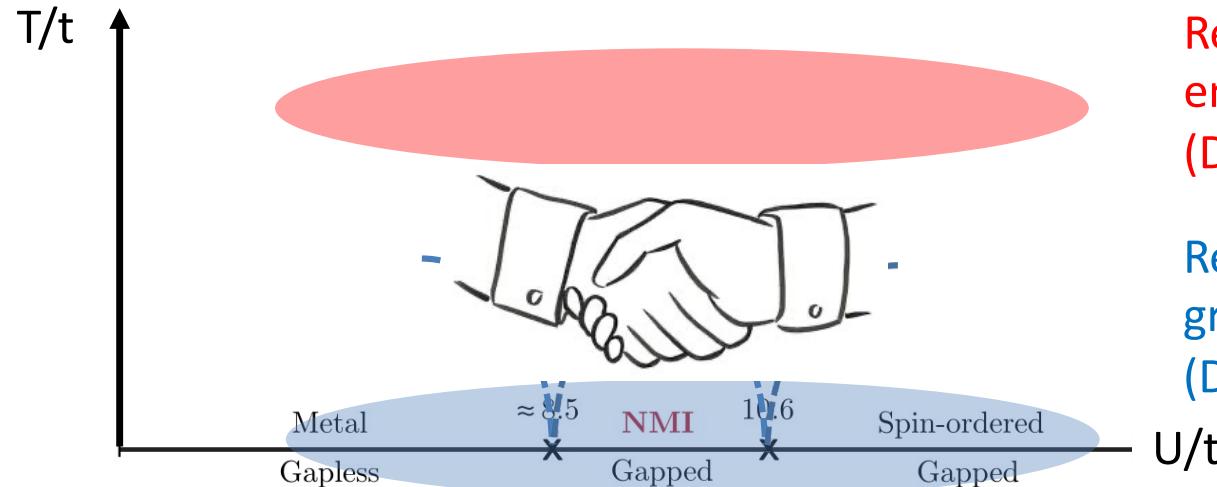
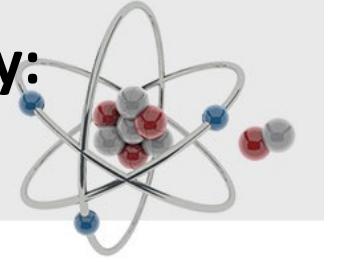
Green function /  
ing methods

(DMFT and extensions, DiagMC)

Realm of wave function /  
ground state methods  
(DMRG, METTS)

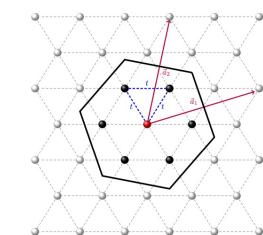
*Y. Shimizu et al., PRL 91, 107001 (2003)*  
*A. Szasz, et al., PRX 10, 021042 (2020)*

# A multimethod, multimesenger study in uncharted territory: The Hubbard model on the triangular lattice



Realm of Green function /  
embedding methods  
(DMFT and extensions, DiagMC)

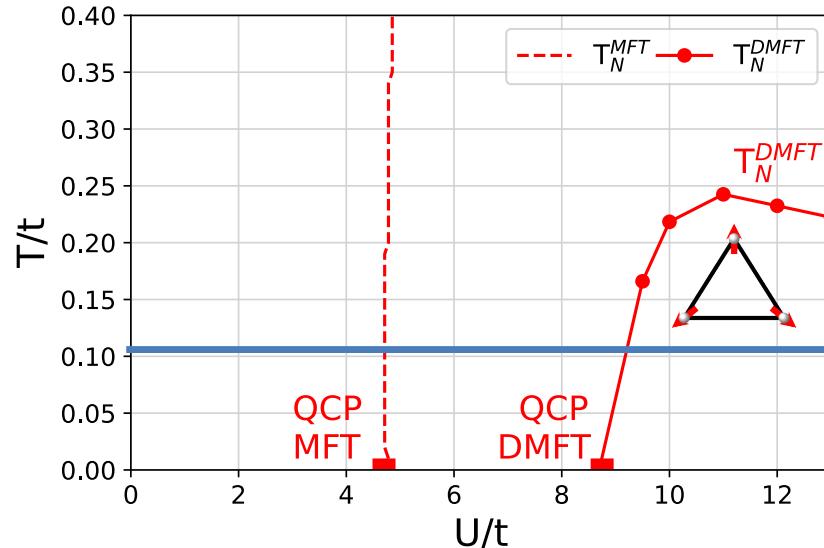
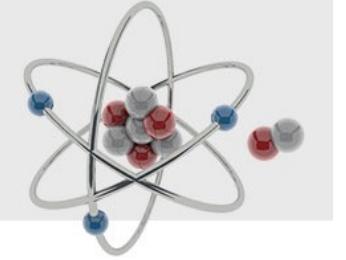
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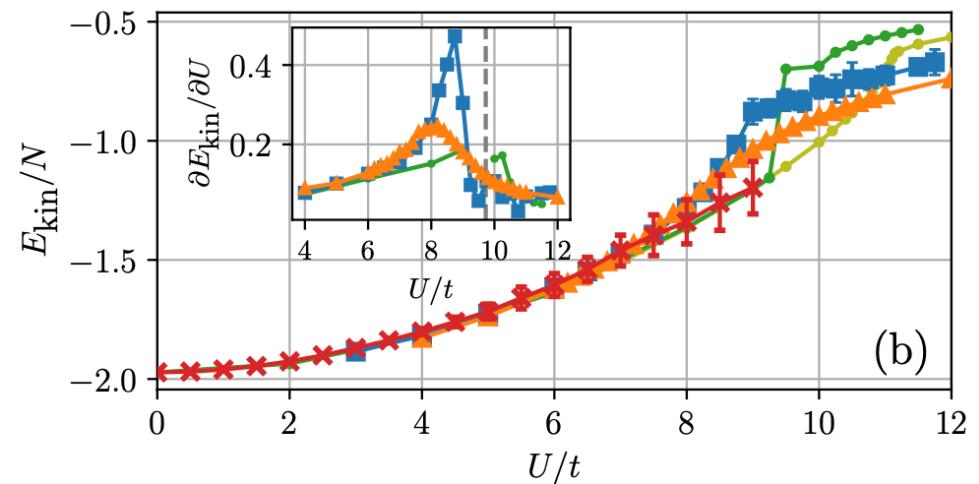
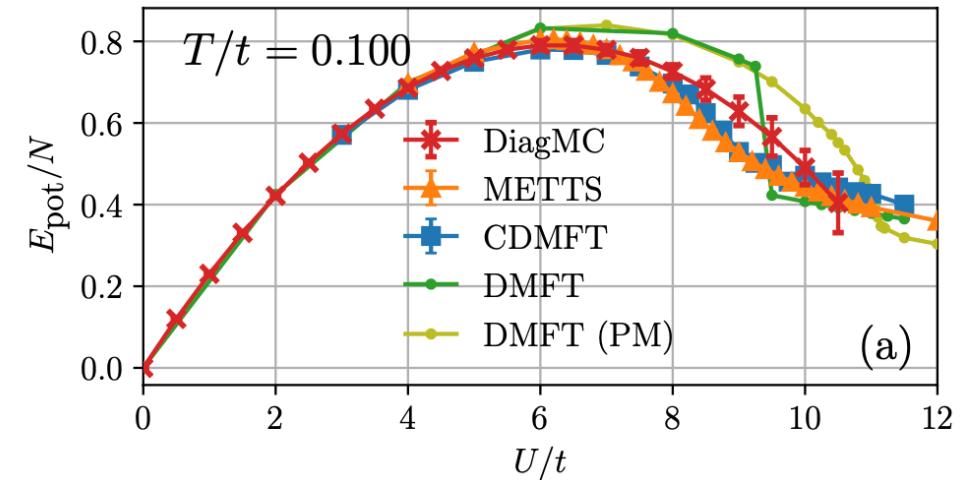
## Methods used in this study:

- Diagrammatic Monte Carlo (DiagMC): benchmark, however limited  $U/T$ -range
- Dynamical mean-field theory (both PM and symmetry-broken)
- Cellular dynamical mean-field theory (CDMFT,  $N_c=7$ ), center-focused
- Minimally entangled thermal typical states (METTS) on  $16 \times 4$  finite-size cylinders  
[S. R. White, Phys. Rev. Lett. **102**, 190601 (2009)]

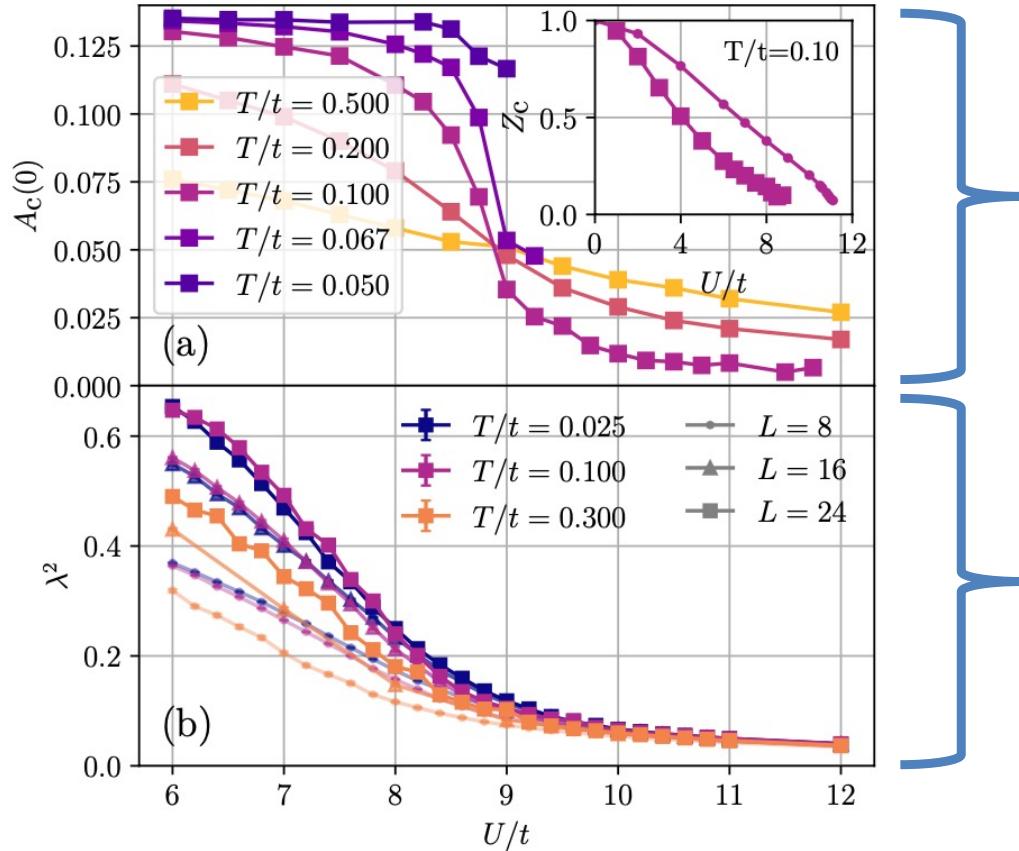
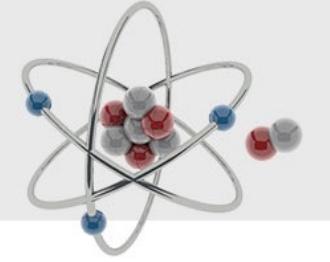
# Establishing common grounds: from the mean-field phase diagram to energetics



- In (D)MFT: transition from PM to 120° Heisenberg magnetism
- All methods agree at low coupling
- METTS and CDMFT show excellent agreement in  $E_{\text{pot}}$  and very good in  $E_{\text{kin}}$



# Metal-insulator crossover



CDMFT: spectral weight

$$A_c(\omega = 0) = -\frac{1}{\pi} \text{Im}G_c(i\omega_n \rightarrow i0^+)$$

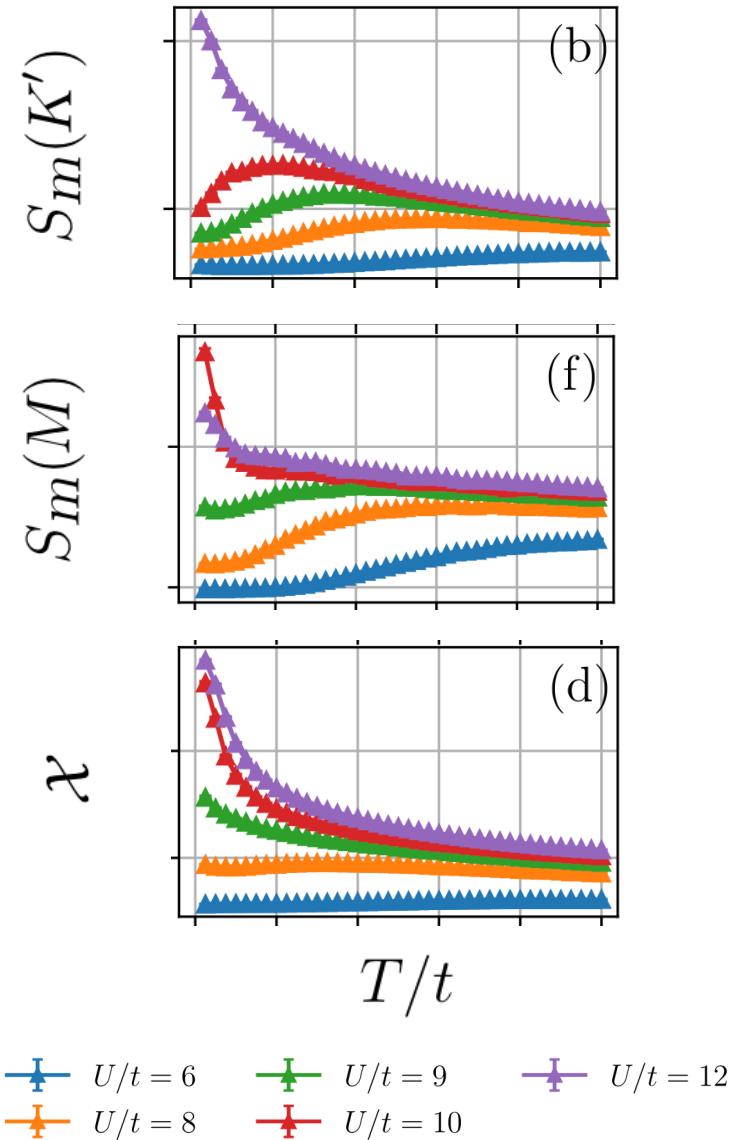
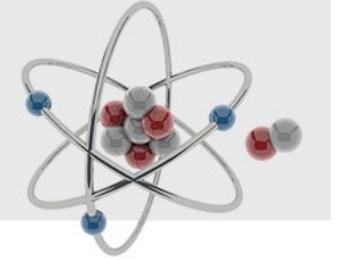
METTS: localization length

$$\lambda^2 = \frac{1}{N} \left( \langle X^2 \rangle - \langle X \rangle^2 \right)$$

Metal-insulator crossover  
at  $U/t \approx 9$   
from CDMFT and METTS

ek, ..., TS and A. Georges, Phys. Rev. X **11**, 041013 (2021)  
H. T. Dang et al., Phys. Rev. B **91**, 155101 (2015)

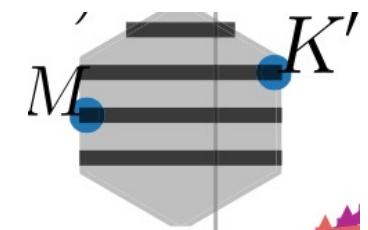
# Magnetism and chirality: a METTS perspective



Magnetic structure factor

$$S_m(\mathbf{k}) = \frac{1}{N} \sum_{l,m=1}^N e^{i\mathbf{k}\cdot(\mathbf{r}_l - \mathbf{r}_m)} \langle \vec{S}_l \cdot \vec{S}_m \rangle$$

Chiral-chiral correlator

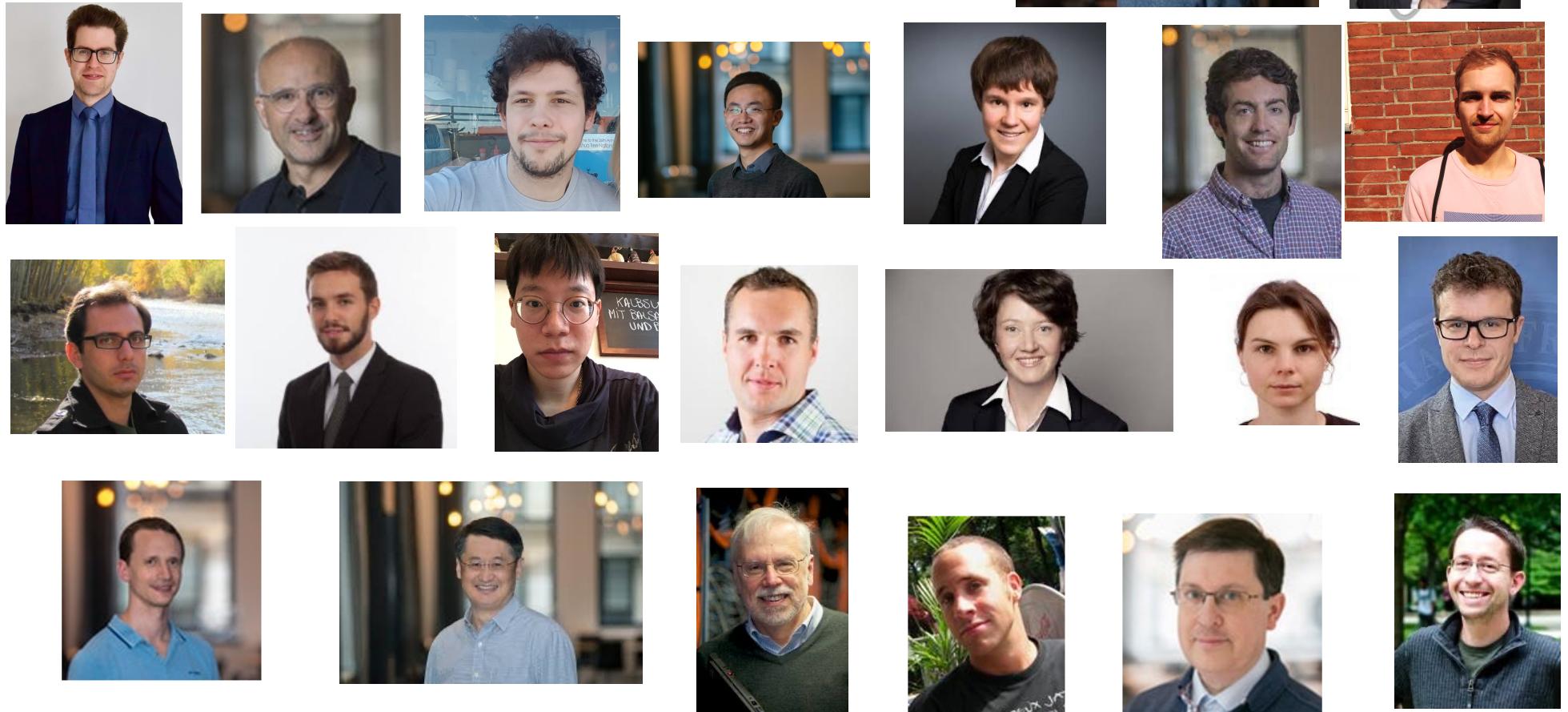


$$\chi = \frac{1}{N} \sum_{\mu, \nu \in \Delta} \langle \chi_\mu \chi_\nu \rangle$$

$$\chi_\mu = \vec{S}_l \cdot (\vec{S}_m \times \vec{S}_n)$$



# Acknowledgements



PHYSICAL REVIEW X 11, 011058 (2021)

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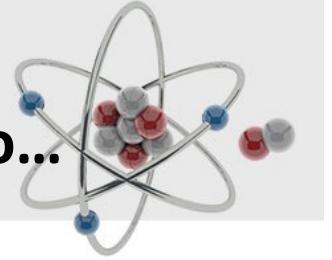
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PHYSICAL REVIEW X 11, 041013 (2021)

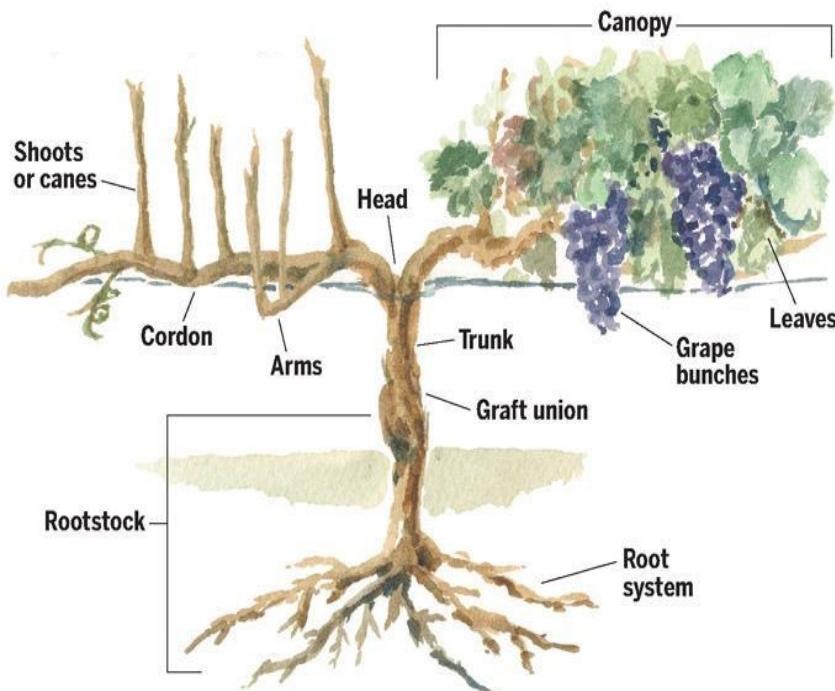
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...real materials



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*Phys. Rev. Research 4, 023093 (2022)*

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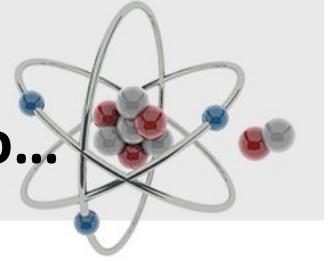
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Real frequency data  
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*Phys. Rev. Research* **4**, 023093 (2022)

- Handshake of Green function and wave-function based methods
- Agreement of CDMFT and METTS
- Intriguing metal-insulator crossovers and magnetic ordering tendencies

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*Physical Review X* **11**, 011058 (2021)

- Extensive comparison of state-of-the-art quantum many-body methods
- Metallic regime with fairly long-ranged correlations
- Non-Fermi-liquid behavior due to perfect nesting