HYDRODYNAMIC DIFFUSION AND ITS BREAKDOWN NEAR AdS_2 QUANTUM CRITICAL POINTS

Richard Davison

Heriot-Watt University

Strange metals: from the Hubbard model to AdS/CFT

Institute of Physics Belgrade, May 2022

2011.12301 [hep-th] / Phys Rev X 11 031024

with Daniel Areán, Blaise Goutéraux, Kenta Suzuki

INTRODUCTION

- The properties of interacting quantum systems at non-zero temperature can be very complicated.

  Especially if there is no quasiparticle or other perturbative description.

- However, some aspects of the dynamics are relatively simple and general.

  Specifically, dynamics of conserved charge density operators over very long time and distance scales.

- These dynamics are governed by simple effective theories: **hydrodynamics**.

  When are these effective theories valid?

HYDRODYNAMICS

- Assumption: local thermal equilibrium is achieved over large enough scales.

$$t \gg \tau_{eq} \qquad x \gg l_{eq}$$

The system can be described by its conserved charge densities.

e.g. energy density $\varepsilon(t, \underline{x})$, charge density $\rho(t, \underline{x})$, momentum density $\overrightarrow{\Pi}(t, \underline{x})$…

- These each obey a local conservation equation: $\partial_t \varepsilon + \nabla \cdot j_\varepsilon = 0$ etc.

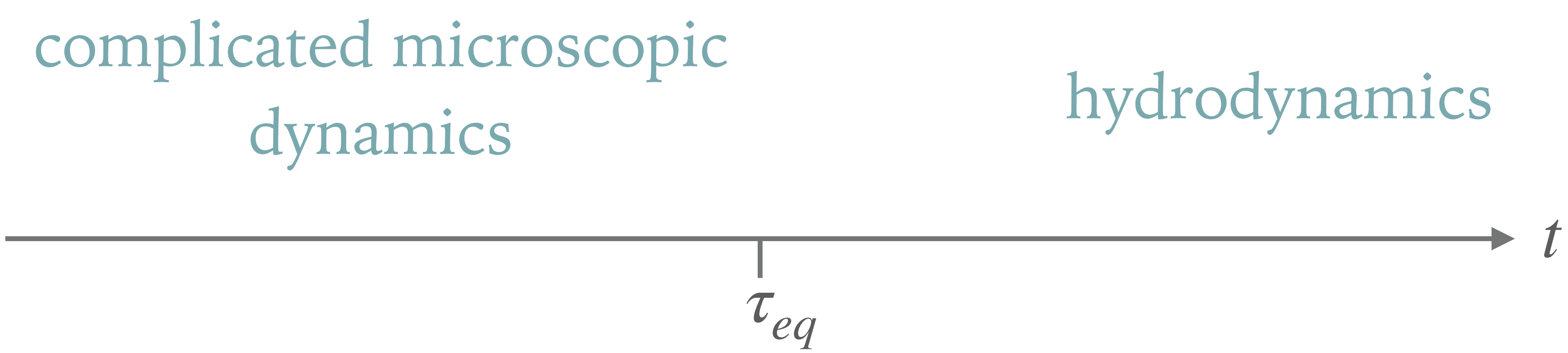
Currents are expressed as a derivative expansion of the densities

e.g. $$j_\varepsilon = -D \nabla \varepsilon - D_2 \nabla^3 \varepsilon - \ldots$$

$\longrightarrow$ Hydrodynamic equations e.g. $$\partial_t \varepsilon = D \nabla^2 \varepsilon + D_2 \nabla^4 \varepsilon + \ldots$$

BREAKDOWN OF HYDRODYNAMICS

- Expect this description to break down at short scales



e.g. in a Fermi liquid $\tau_{qp} \sim 1/T^2$ and so expect $\tau_{eq} \sim 1/T^2$

- If there are no quasiparticles, we would perhaps expect $\tau_{eq} \sim 1/T$. Can we say anything more than this?

- Yes: local equilibration scales τ_{eq} and l_{eq} are quantitatively related to basic properties of the strange metal state.

LOCAL EQUILIBRATION SCALES

- τ_{eq} and l_{eq} are roughly the scales at which non-hydro degrees of freedom become important.

- Diagnose from Green's functions of conserved charge densities (e.g. $G_{\varepsilon\varepsilon}(\omega, k)$)

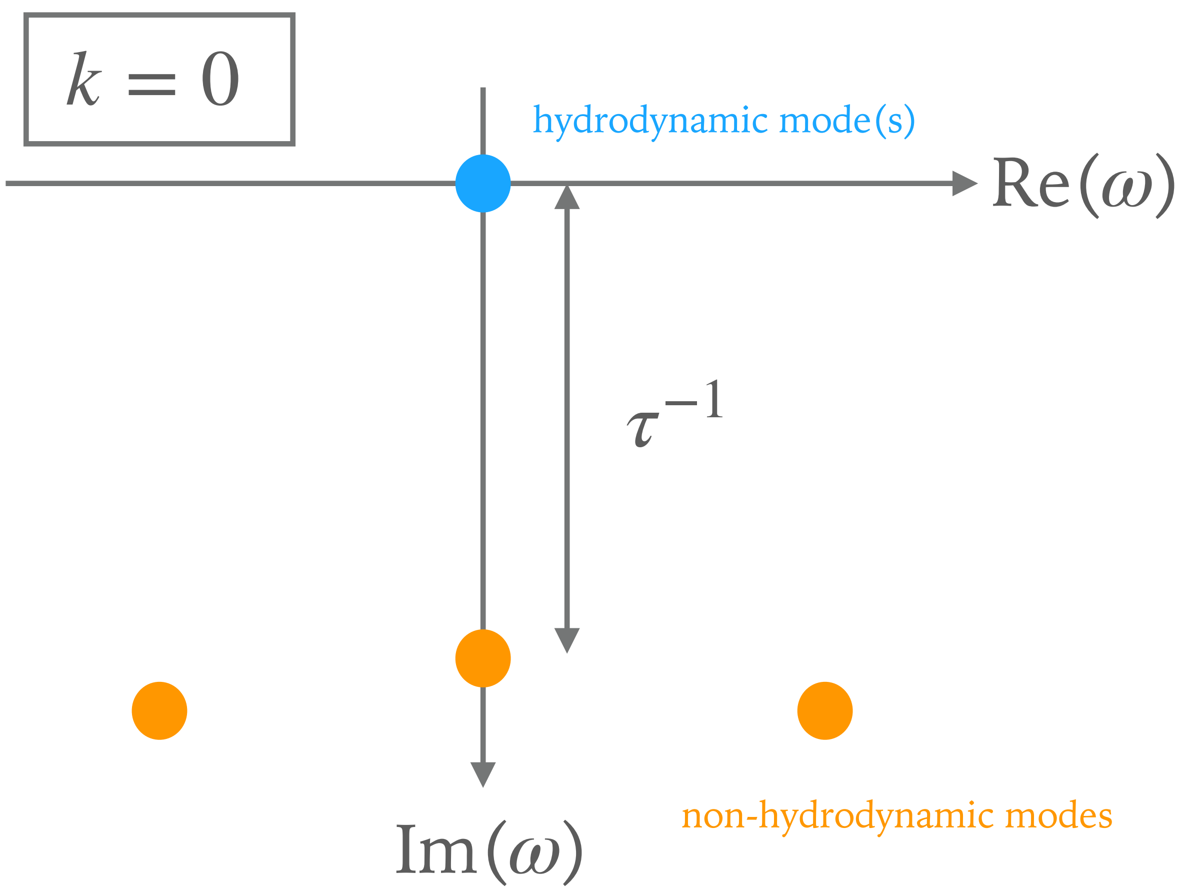
  * hydrodynamic modes: gapless

    e.g. $\partial_t \varepsilon = D\,\nabla^2 \varepsilon + D_2\,\nabla^4 \varepsilon + \ldots$

    $\omega_{hydro}(k) = -\,iDk^2 - iD_2 k^4 + \ldots$

  * non-hydrodynamic modes: gapped

    e.g. $\omega_{non-hydro}(k) = -\,i\tau^{-1} + \ldots$



CONVERGENCE OF DISPERSION RELATIONS

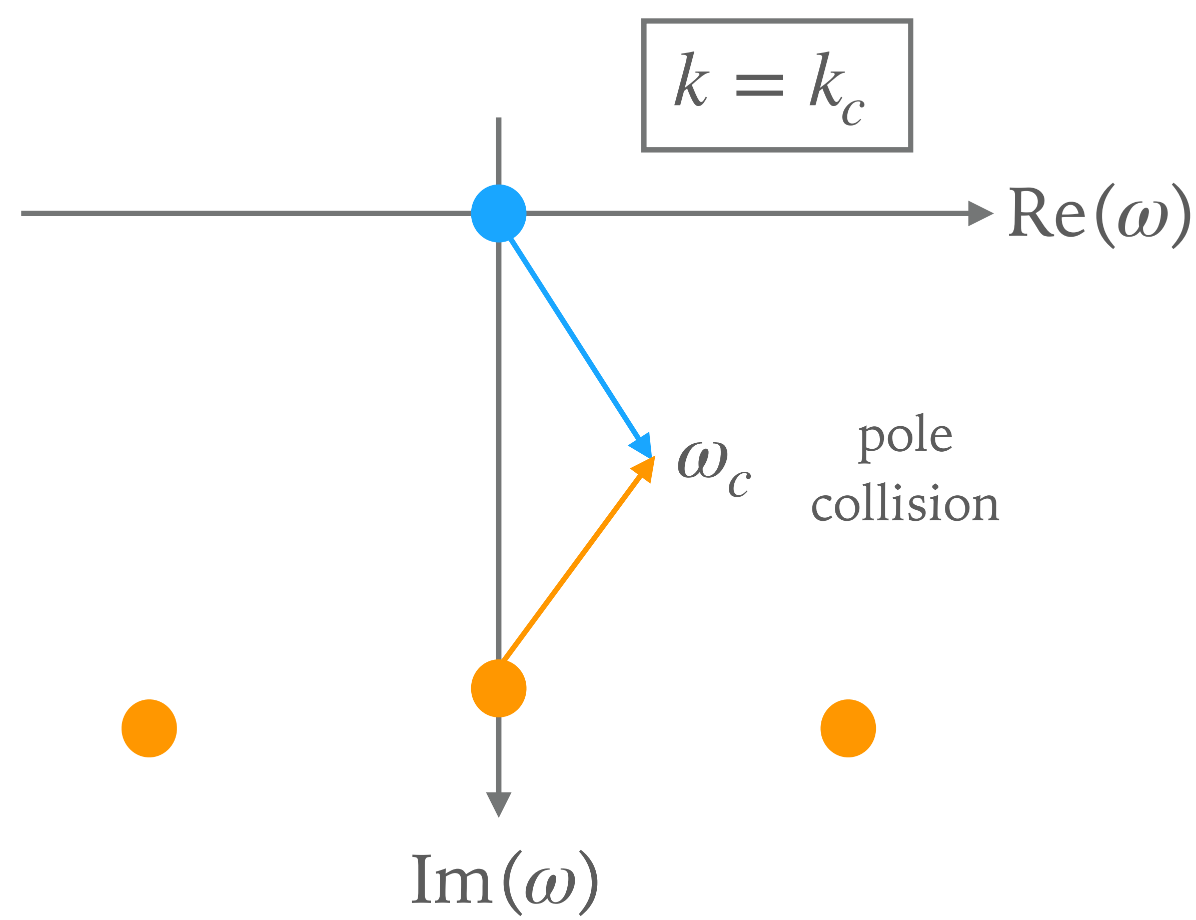
- More precise definition: use the radius of convergence k_{eq} of the hydro series

$$\omega_{hydro}(k) = -i \sum_{i=1}^{\infty} D_n k^{2n}$$

- k_{eq} is sensitive to properties of the non-hydrodynamic modes:

$$l_{eq} = \frac{1}{k_{eq}} = \frac{1}{|k_c|}$$

$$\tau_{eq} = \frac{1}{\omega_{eq}} = \frac{1}{|\omega_c|}$$



Withers ;
Grozdanov, Kovtun, Starinets & Tadic

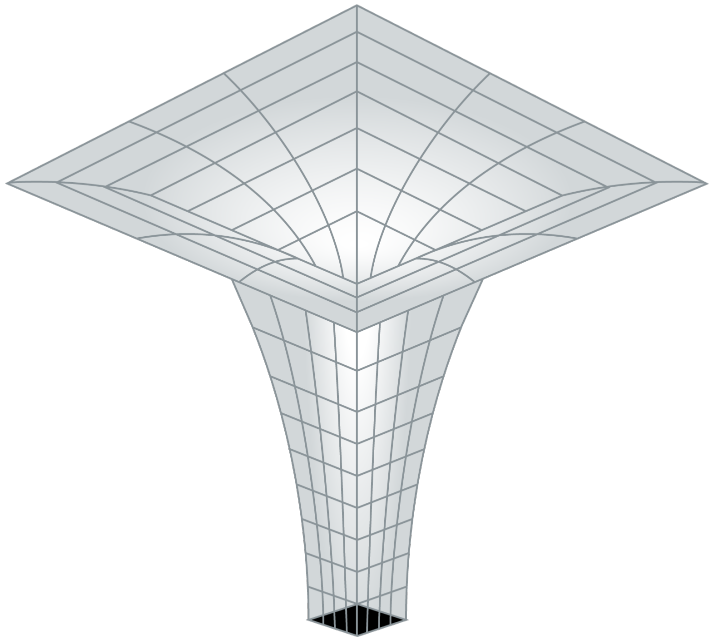
THEORIES WITH STRANGE METAL STATES

- Look at strange metal states for which we have a microscopic theory:
  - ✳ Quantum field theories with a holographic description

High energy: conformal field theory

RG flow

Low energy: quantum critical state



local criticality

$(t, \underline{x}) \to (\lambda t, \underline{x})$

$\longleftrightarrow$

$AdS_2 \times R^d$ metric in interior

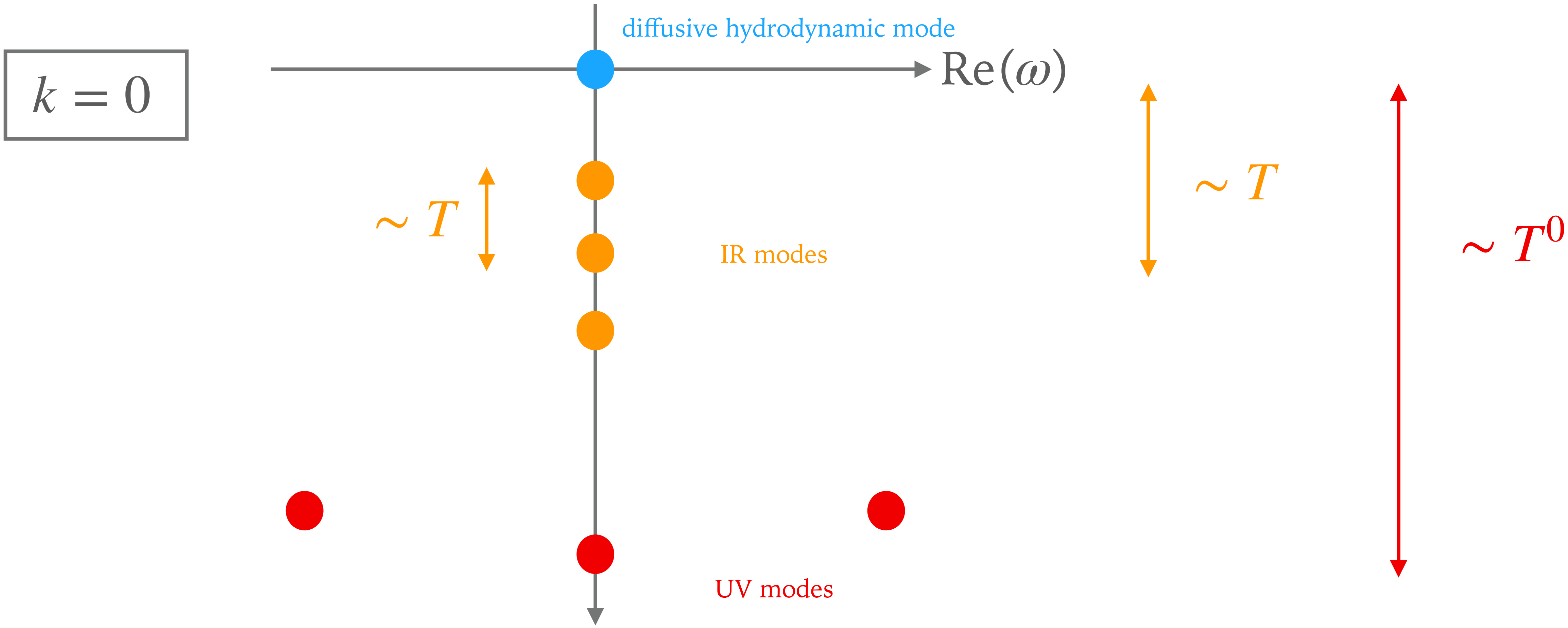
$$ds^2 = -\frac{r^2}{L^2}dt^2 + \frac{L^2 dr^2}{r^2} + L_x^2 d\underline{x}_d^2$$

- ✳ SYK-like models of interacting fermions

STRUCTURE OF GREEN'S FUNCTIONS

- Studied two holographic examples, with very different hydrodynamics:
  - ✳ Linear axion: energy density diffuses
  - ✳ Reissner-Nordstrom: momentum density and temperature diffuse

- At low T, Green's function poles have a similar structure in all cases



THE IR MODES

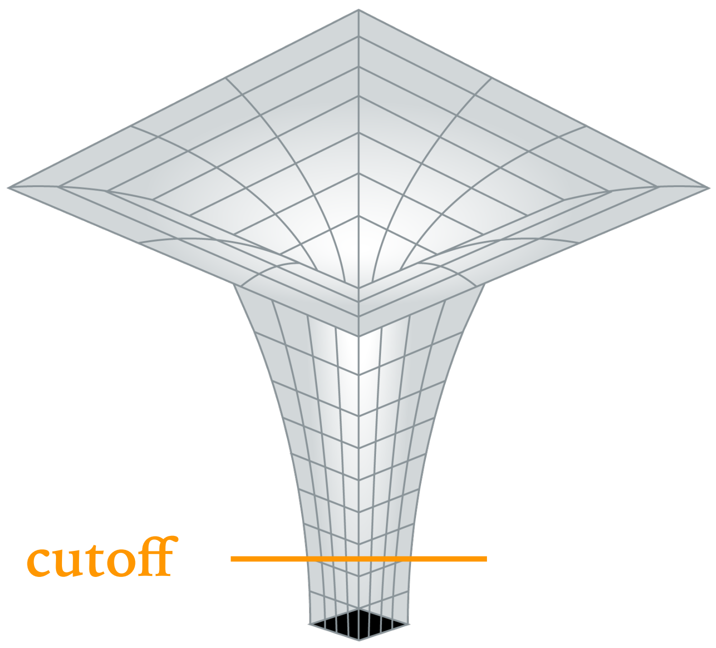
- The IR modes control the breakdown of hydrodynamics.

  These are collective excitations of the locally critical degrees of freedom.

- Artificially remove all other degrees of freedom by cutting off the spacetime, keeping only the $AdS_2 \times R^d$ region.

  Each Green's function is characterised by a scaling dimension $\Delta(k)$:

$$\mathcal{G}_{IR}(\omega, k) \propto T^{2\Delta(k)-1} \frac{\Gamma\left(\frac{1}{2} - \Delta(k)\right)\Gamma\left(\Delta(k) - \frac{i\omega}{2\pi T}\right)}{\Gamma\left(\frac{1}{2} + \Delta(k)\right)\Gamma\left(1 - \Delta(k) - \frac{i\omega}{2\pi T}\right)}$$

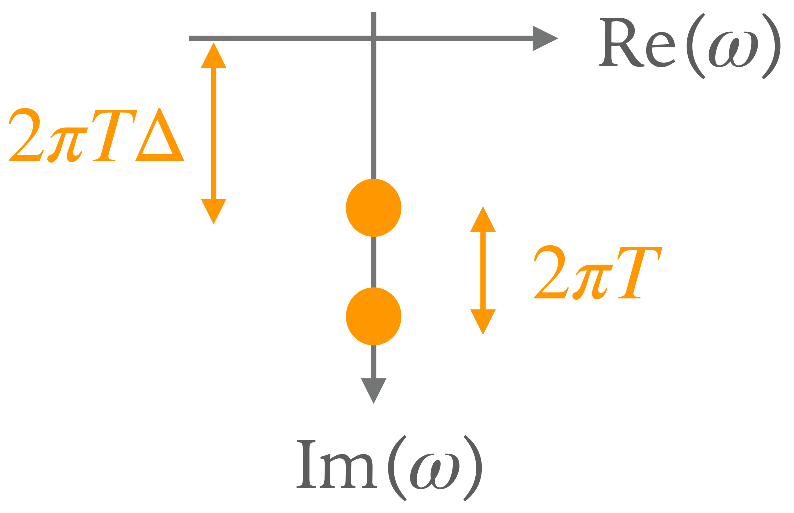


Faulkner, Liu, McGreevy & Vegh

THE IR MODES

- The locally critical Green's function has poles at

$$\omega_n(k) = -i2\pi T(n + \Delta(k)) \qquad n = 0,1,2,\ldots$$



- At small T and k the full Green's function inherits these poles: the IR modes.

- In the examples we studied:

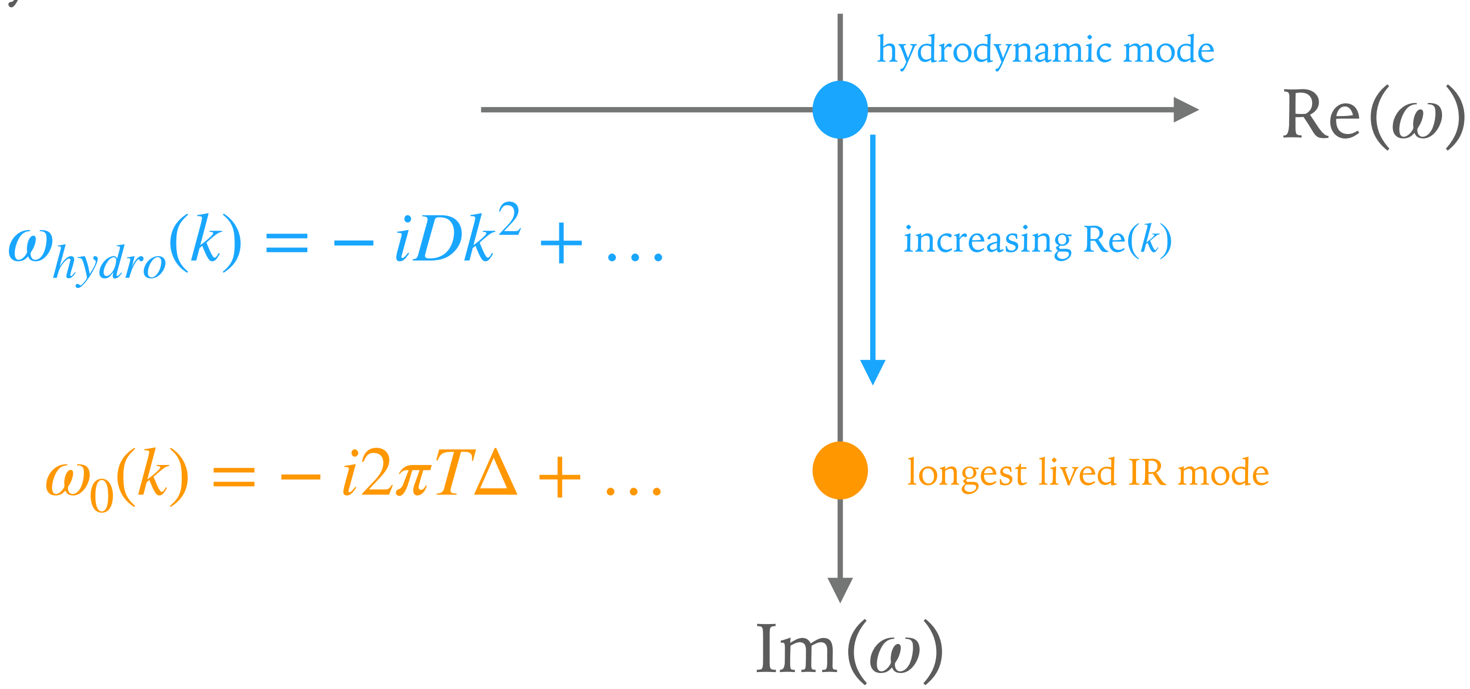
  ✳ Linear axion: $2\Delta(k) = 1 + \sqrt{9 + 8k^2/k_L^2} \longrightarrow \omega_n(0) = -i2\pi T(n+2)$

  ✳ Reissner-Nordstrom: $2\Delta_T(k) = 1 + \sqrt{5 + 8k^2/\mu^2 + 4\sqrt{1 + 4k^2/\mu^2}} \longrightarrow \omega_n(0) = -i2\pi T(n+2)$

  Edalati, Jottar & Leigh $2\Delta_\Pi(k) = 1 + \sqrt{5 + 8k^2/\mu^2 - 4\sqrt{1 + 4k^2/\mu^2}} \longrightarrow \omega_n(0) = -i2\pi T(n+1)$

BREAKDOWN OF HYDRODYNAMICS: EXPECTATIONS

- Expect locally critical degrees of freedom to be responsible for the breakdown of hydrodynamics



- Be very naive: the modes will collide at

$$\omega_c = -i2\pi T\Delta \qquad \text{and} \qquad k_c^2 = \frac{2\pi T\Delta}{D}$$

assuming all corrections to dispersion relations are negligible for $k < k_c$

BREAKDOWN OF HYDRODYNAMICS: RESULTS

- For $k < k_c$, the corrections **are** parametrically small in the low T limit!
- In the low T limit

$$\omega_{eq} \to 2\pi T\Delta \qquad k_{eq}^2 \to \frac{\omega_{eq}}{D} = \frac{2\pi T\Delta}{D}$$

Δ : scaling dimension

D : hydrodynamic diffusivity

- Local equilibration scales are governed by the low energy properties Δ and D.
- Conversely, the local equilibration scales set the value of the diffusivity:

$$D = \omega_{eq} k_{eq}^{-2}$$

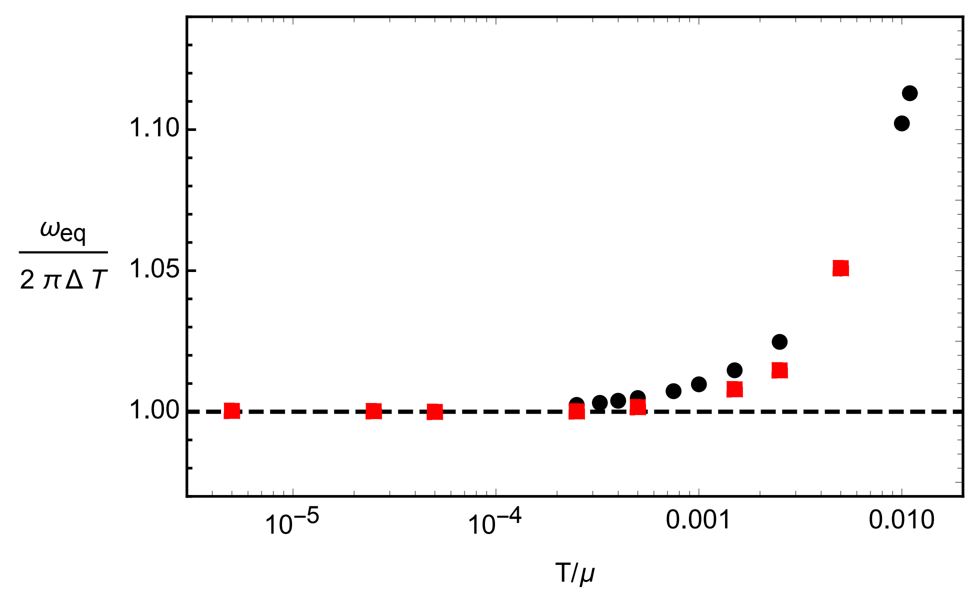
EXPLICIT HOLOGRAPHIC EXAMPLES

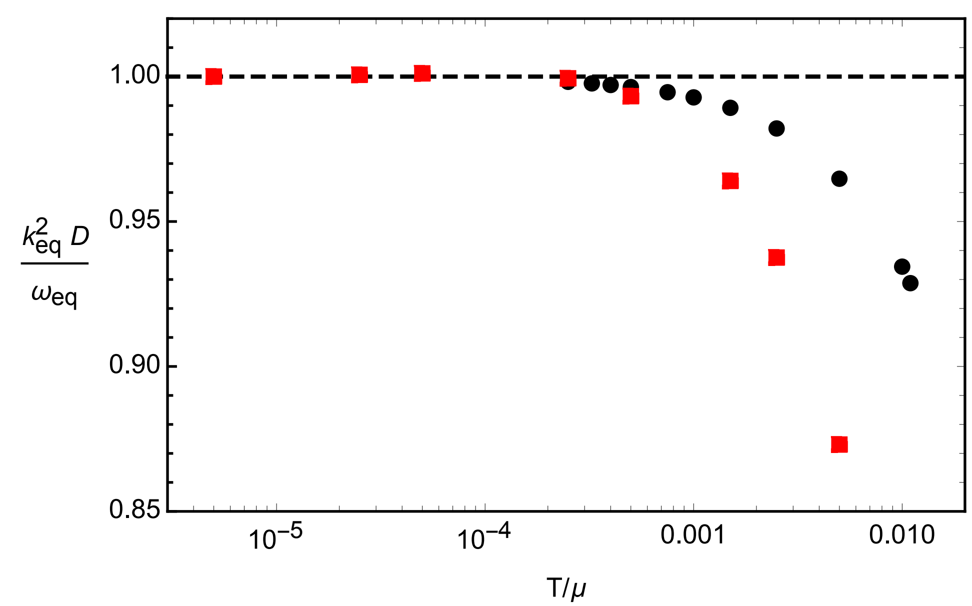
- Analytic results in the linear axion case:

$$\omega_{eq} = 2\pi T\Delta\left(1 + \frac{8\sqrt{6}\pi T}{9k_L} + \dots\right) \qquad k_{eq}^2 = \frac{\omega_{eq}}{D}\left(1 - \frac{4\sqrt{6}\pi T}{3k_L} + \dots\right)$$

- Numerical results in the Reissner-Nordstrom cases:

● diffusion of temperature perturbations
■ diffusion of momentum perturbations





consistent with Withers ; Jansen & Pantelidou ; Abassi & Tahery

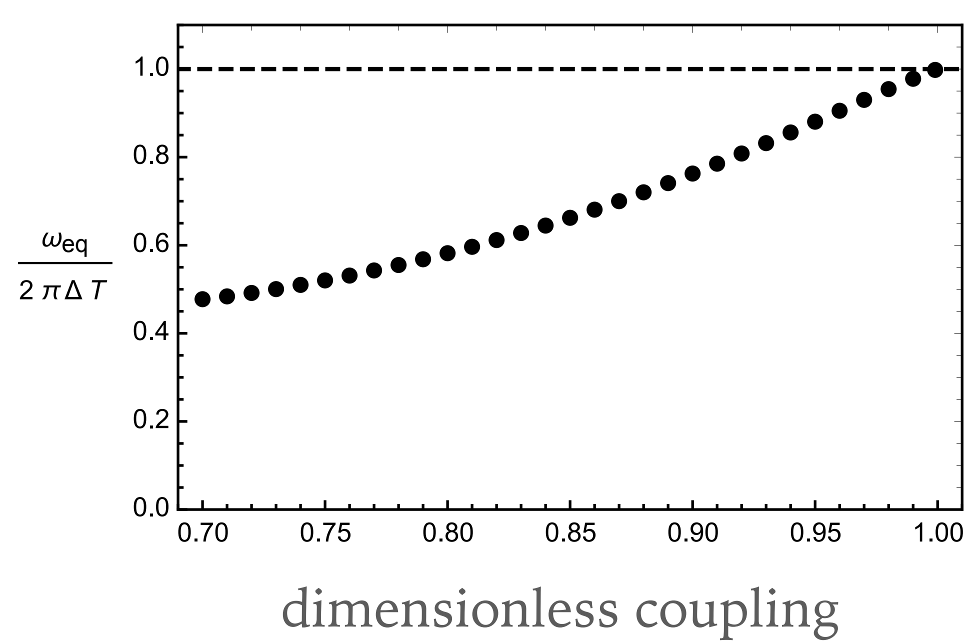
SYK CHAIN MODEL

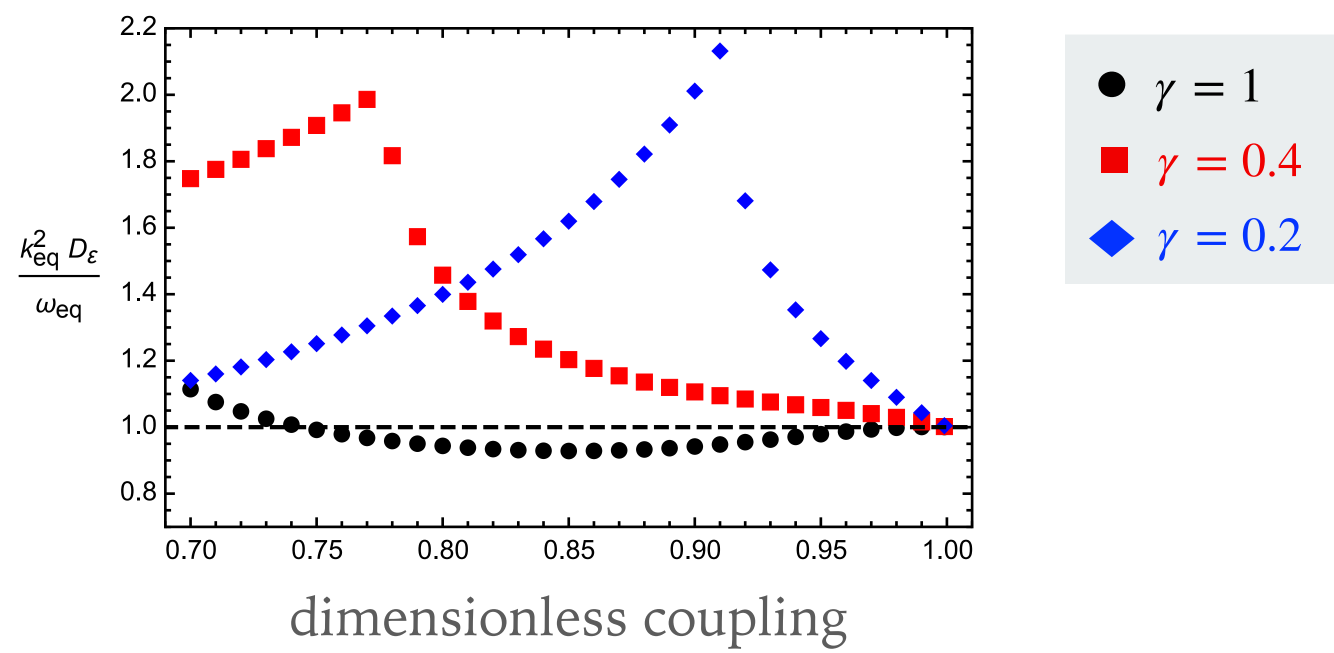
- SYK chain model is a theory of N interacting fermions with local criticality.

$$H = i^{q/2} \sum_{x=0}^{M-1} \left(\sum_{1 \le i_1 < \ldots < i_q \le N} J_{i_1 \ldots i_q, x} \chi_{i_1,x} \cdots \chi_{i_q,x} + \sum_{\substack{1 \le i_1 < \ldots < i_{q/2} \le N \\ 1 \le j_1 < \ldots < j_{q/2} \le N}} J'_{i_1 \ldots i_{q/2} j_1 \ldots j_{q/2}, x} \chi_{i_1,x} \cdots \chi_{i_{q/2},x} \chi_{j_1,x+1} \cdots \chi_{j_{q/2},x+1} \right),$$

There is one hydrodynamic mode: diffusion of energy. Gu, Stanford & Qi

- Exact calculation of $G_{\varepsilon\varepsilon}(\omega, k)$ for $N \gg q^2 \gg 1$. Choi, Mezei & Sárosi





SUMMARY

- Looked at states governed by AdS_2 quantum critical points at low energy.

  Simple relations between equilibration timescales and low energy properties

$$\omega_{eq} \to 2\pi T\Delta \qquad\qquad k_{eq}^2 \to \frac{\omega_{eq}}{D} = \frac{2\pi T\Delta}{D}$$

  Other examples: Liu & Wu ; Huh, Jeong, Kim & Sun ; Wu, Baggioli & Li

- These simple relations are a consequence of two properties
  - ✳ Longest-lived non-hydrodynamic mode is a collective excitation of the locally critical degrees of freedom ; lifetime set by scaling dimension Δ .
  - ✳ Corrections to the quadratic approximation to the hydrodynamic dispersion relation are parametrically small for $k < k_c$.

OPEN QUESTIONS

- Why does the quadratic approximation to hydrodynamics work so well?!

- Use universality of non-hydro modes to constrain transport coefficients?

- Generalisations:

  ✴ other AdS_2 fixed points (reason to be confident, at least in some cases)
  ✴ AdS_2 with non-universal deformation
  ✴ non-AdS_2 fixed points (e.g. Lifshitz)
  ✴ multiple diffusion modes in one Green's function (e.g. complex SYK chain)

THANK YOU!