Spectral functions in holographic lattices and the Hubbard model

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Key2SM

SCIENTIFIC COMPUTING LABORATORY

Outline

- Why Hubbard model?!
- The basic holographic model: spectral functions and the fit to the spectral functions of the Hubbard model
- Variations of the model and universality
- What have we learned?

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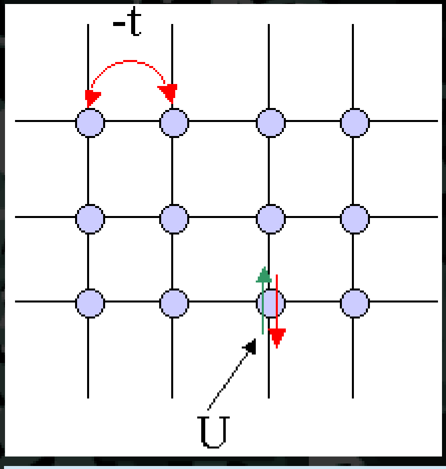
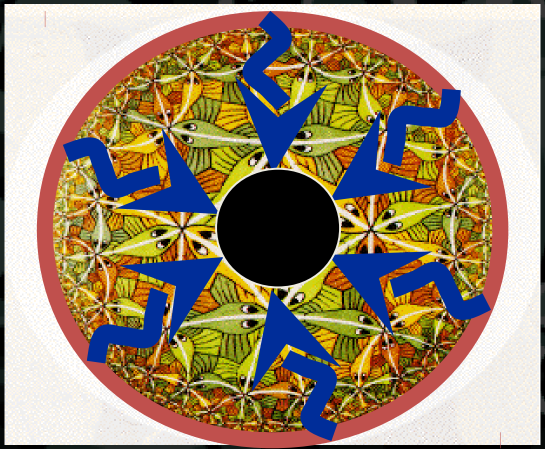
Hubbard model vs. holography



$$H = -t \sum_{\langle ij \rangle;\sigma} c^{+}_{i\sigma} c_{j\sigma} + \text{h.c.} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

U(1)-charged electrons, pairwise interactions



No Hamiltonian known
SU(N)-charged bosons and fermions in the large-N limit

- At first glance rather hopeless and poorly motivated
- Beware of large N pathologies

Hubbard model vs. holography

- Holographic SU(N) Bose-Hubbard model constructed in 1411.7899 (Fujita, Harrison, Karch, Meyer & Paquette 2014)
- Idea: quiver gauge theory on hard-wall AdS provides a microscopic dual of the Hubbard lattice

Rene Meyer

Andreas Karch

- Our idea: brute force search of bottom-up effective holographic theories

The roadmap

- Empirical motivation:

Some of the bottom-up holographic spectra look very much like the Hubbard model Quantum Monte Carlo results! It's worth trying!

- Our idea: brute force search in the parameter space of a phenomenological (bottom-up) Einstein-Maxwell-dilaton model
- The citerion: spectral function in Matsubara frequencies

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The holographic model

- Einstein-Maxwell-dilaton + 2D "ionic lattice"

$$S=\int d^4x\sqrt{-g}\left[R-(\nabla\varphi)^2-\frac{Z(\varphi)}{4}F^2-V(\varphi)\right]$$

Ionic square lattice: modulating chemical potential

$$\mu(x,y)=A_t(\mathrm{AdS\ bnd})=\mu_0+\delta\mu\cos\pi x\cos\pi y$$

- Dynamical UV cutoff: impose the ARPES sum rules through a frequency-dependent "double-trace" deformation
- Bulk dipole coupling to electric field

$$\left(D_\mu\Gamma^\mu-m-ipF_{\mu\nu}\Gamma^{\mu\nu}\right)\Psi=0$$

Effectively shifts momentum and mimics a gap

EMD action

- Einstein-Maxwell-dilaton + 2D "ionic lattice"

$$S=\int d^4x\sqrt{-g}\left[R-(\nabla\varphi)^2-\frac{Z(\varphi)}{4}F^2-V(\varphi)\right]$$

$$Z(\varphi)=\cosh(2\alpha\varphi),\quad V(\varphi)=2V_0\cosh(2\delta\varphi)$$

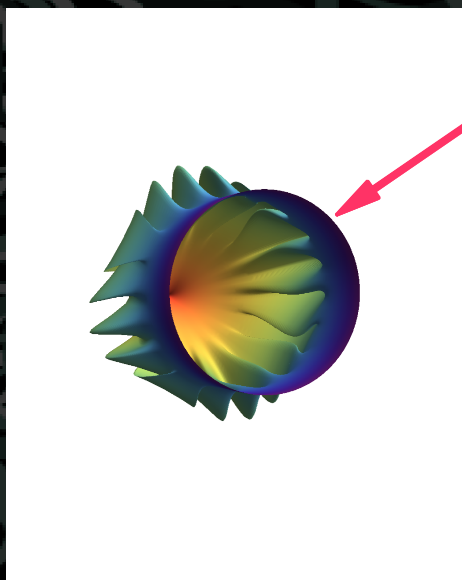
- Potentials from Iizuka et al 2011 [1105.1162], a special case of effective holographic theories [1005.4690, 1107.2116] by Kiritsis, Gouteraux, Meyer...
- Weak lattices – good control, solution for Hubbard:

$$\mu(x,y)=A_t(\text{AdS bnd})=\mu_0+\delta\mu\cos\pi x\cos\pi y,\quad \delta\mu/\mu_0\leq 1$$

- Strong lattices – exciting phenomenology but more work is needed!

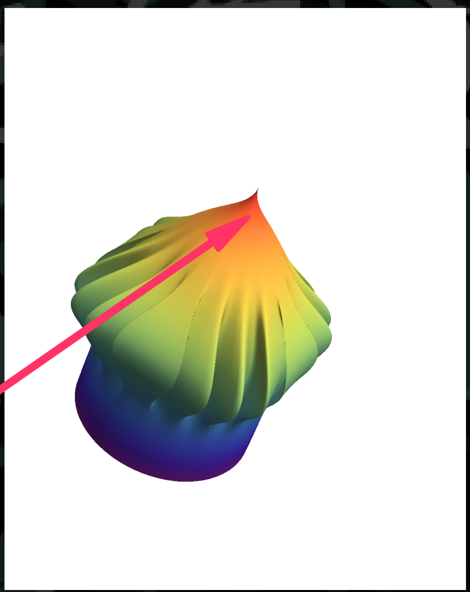
$$\mu(x,y)=A_t(\text{AdS bnd})=\mu_0+\delta\mu\cos\pi x\cos\pi y,\quad \delta\mu/\mu_0>1$$

Corrugated EMD black hole



AdS asymptotics
near the boundary

Zero-T horizon



- Collocation 3D grid solver with Gauss-Lobato basis in z and Fourier basis in x,y + Broyden's iterative nonlinear solver (e.g. book by John Boyd; notes by Krikun [1801.01483]

Dynamical cutoff

- ARPES sum rules violated by holographic spectral functions – UV is determined by conformal dimension $G(\omega,k)\sim\omega^{2\Delta}$
- Idea: double-trace deformation with ω-dependent source (Gursoy et al 1112.5074)
- Holographic realization: obtain the source as the Green function of another holographic fermion in hard-wall AdS (see also Fujita et al 0810.5394)

$$\tilde{G}_R(\omega,\boldsymbol{k})=\left(g_{\text{hard wall}}(\omega,k)-G_R(\omega,\boldsymbol{k})\right)^{-1}$$

- Requires alternative quantization so we need $-1/2<m<1/2$

The algorithm

(1) compute a (α,δ) grid of holographic lattice backgrounds

(2) compute (real-frequency) EDCs for a range of conformal dimensions Δ for each background

(3) translate the holographic EDCs into Matsubara frequencies

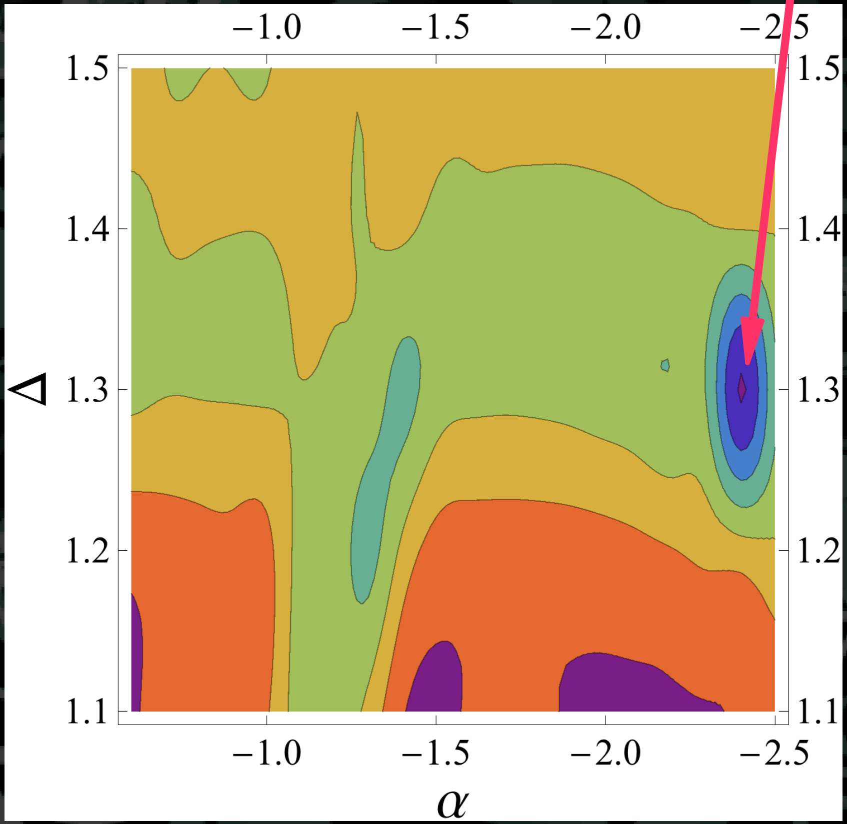
(4) do a grid search in $(\alpha,\delta;\Delta)$ for the best fit to the CTINT Hubbard model Matsubara EDCs with the merit function:

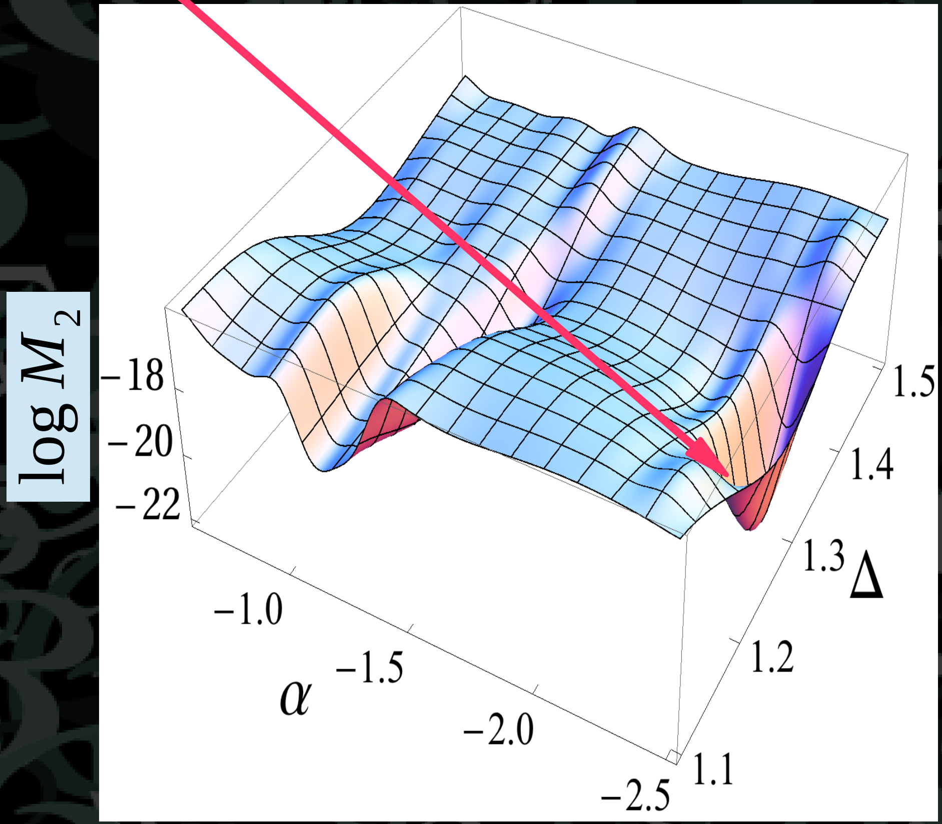
$$M_d \equiv \frac{1}{N^d} \sum_{n,j} \left| G_{\mathrm{AdS}}(i\omega_n;\boldsymbol{k}_j) - G_{\mathrm{CTINT}}(i\omega_n;\boldsymbol{k}_j) \right|^d = \min$$

- similar results for $d=-2,-1,+1,+2$
- (5) repeat for several temperatures and densities

Minimizing the merit function

Global minimum of M_2 – the solution

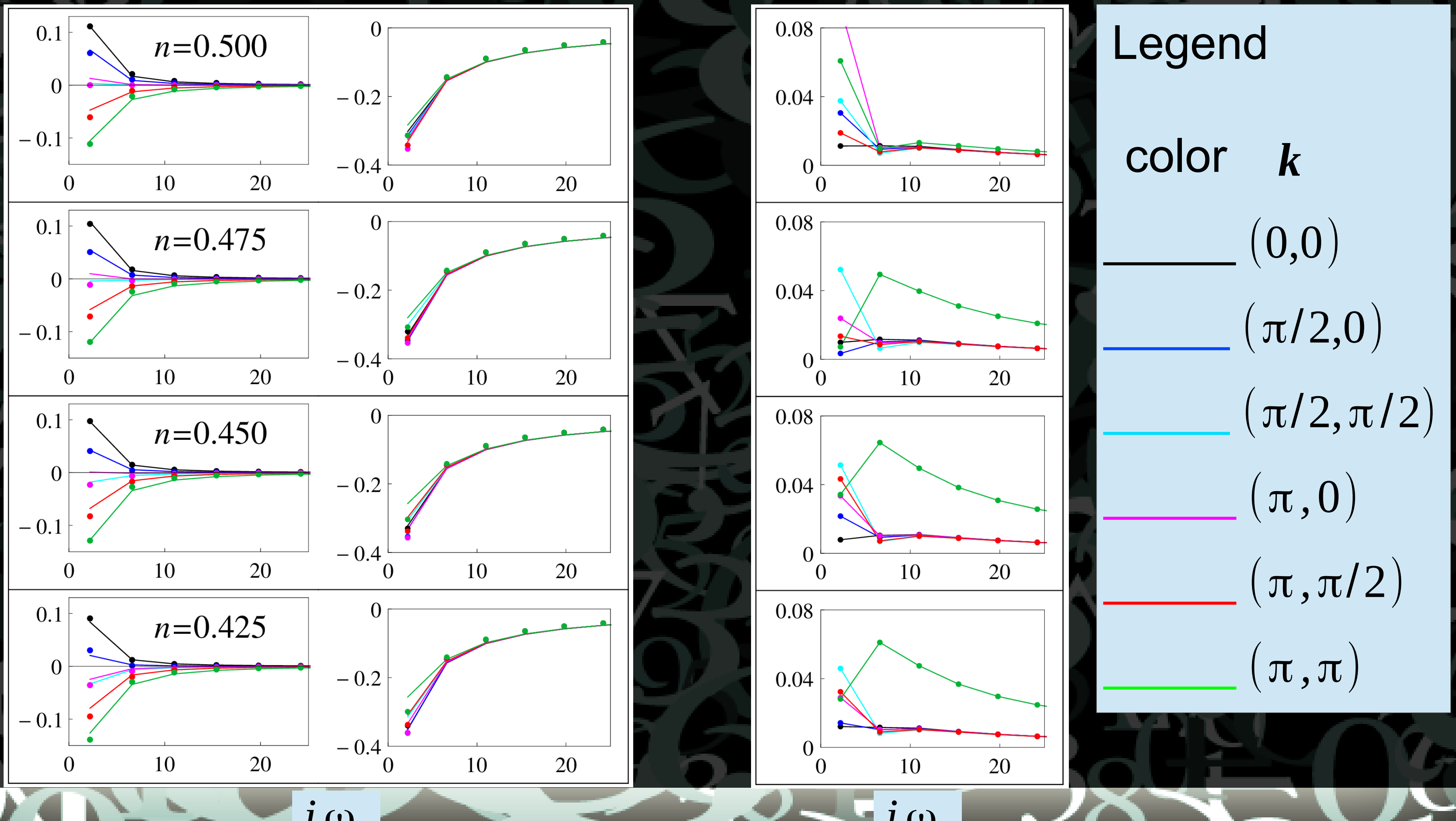




And the outcome is ...

Solution: $\alpha=-1.8, \ \delta=1.0, \ \Delta=1.2$, here for $T=0.70$

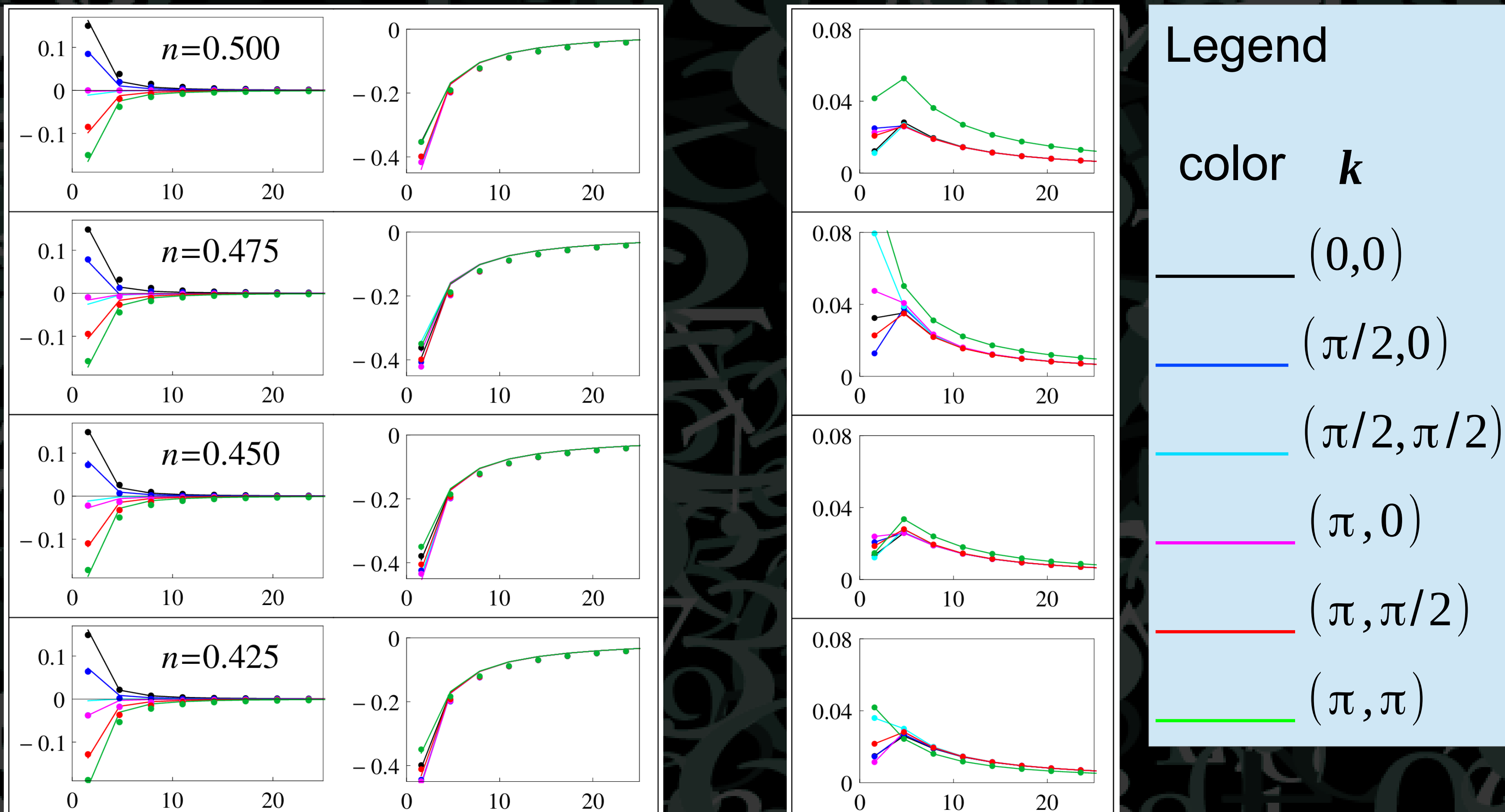
$\mathrm{Re}\, G(i\omega_n, \boldsymbol{k})$ $\mathrm{Im}\, G(i\omega_n, \boldsymbol{k})$ $M_1(i\omega_n, \boldsymbol{k})$



And the outcome is ...

Solution: $\alpha=-1.8, \quad \delta=1.0, \quad \Delta=1.2$, here for $T=0.50$

$\mathrm{Re}\, G(i\omega_n, \boldsymbol{k})$ $\quad$ $\mathrm{Im}\, G(i\omega_n, \boldsymbol{k})$ $\quad$ $M_1(i\omega_n, \boldsymbol{k})$

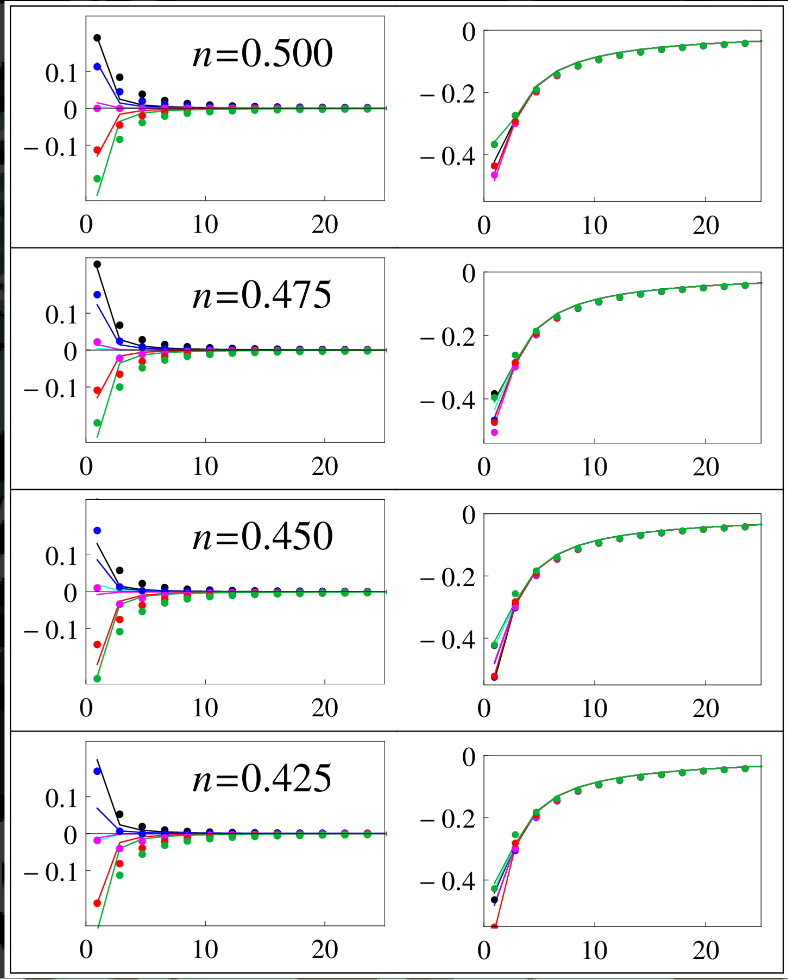


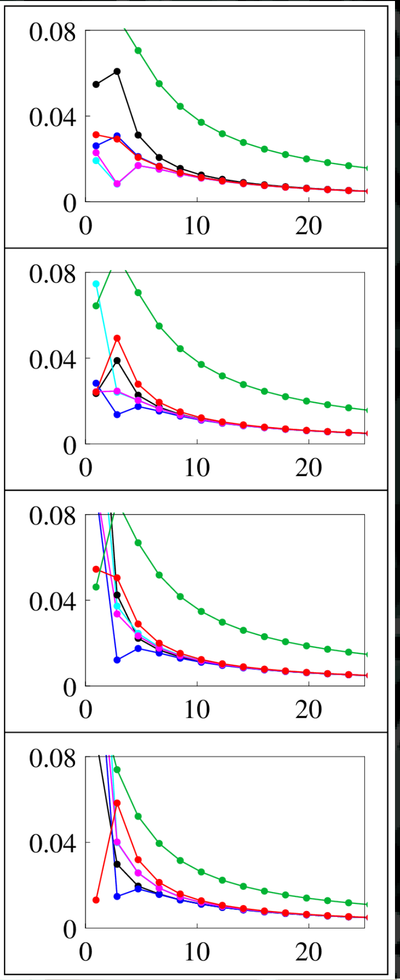
$i\omega_n$ $\quad$ $i\omega_n$

And the outcome is ...

Solution: $\alpha = -1.8, \ \delta = 1.0, \ \Delta = 1.2$, here for $T = 0.30$

$\mathrm{Re}\, G(i\omega_n, \boldsymbol{k})$ $\mathrm{Im}\, G(i\omega_n, \boldsymbol{k})$ $M_1(i\omega_n, \boldsymbol{k})$





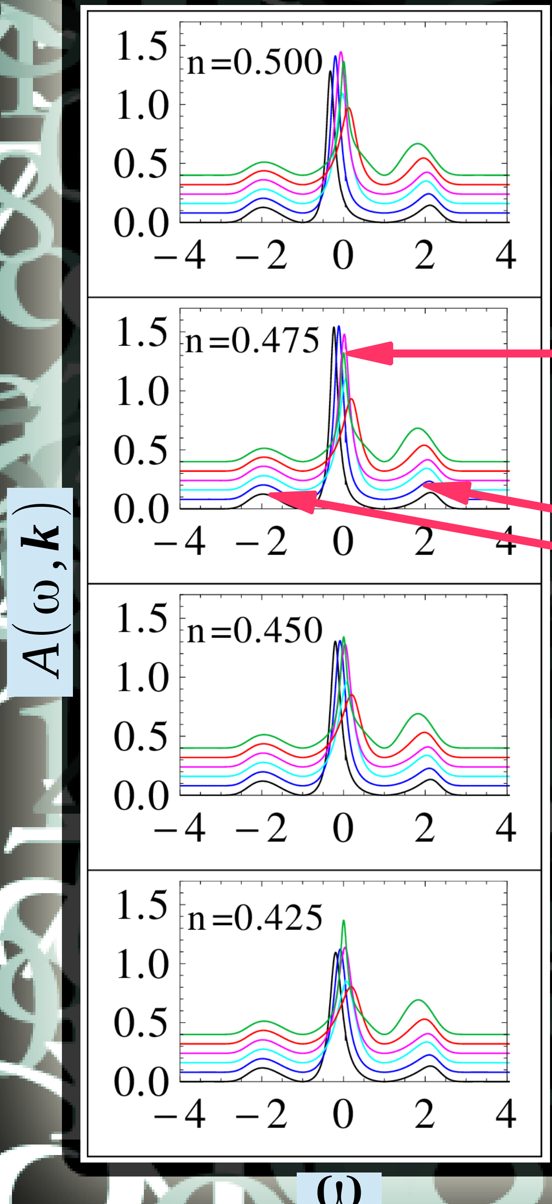
$i\omega_n$ $i\omega_n$

Legend

color	$\boldsymbol{k}$
black	$(0,0)$
blue	$(\pi/2,0)$
cyan	$(\pi/2,\pi/2)$
magenta	$(\pi,0)$
red	$(\pi,\pi/2)$
green	(π,π)

Real-frequency spectra

Same parameters as before @ $T = 0.50$



Broad peak, not really a quasiparticle

Well-separated Hubbard bands

This is the Fermi-liquid-like phase with well-defined QP

Legend

color	$\boldsymbol{k}$
black	$(0,0)$
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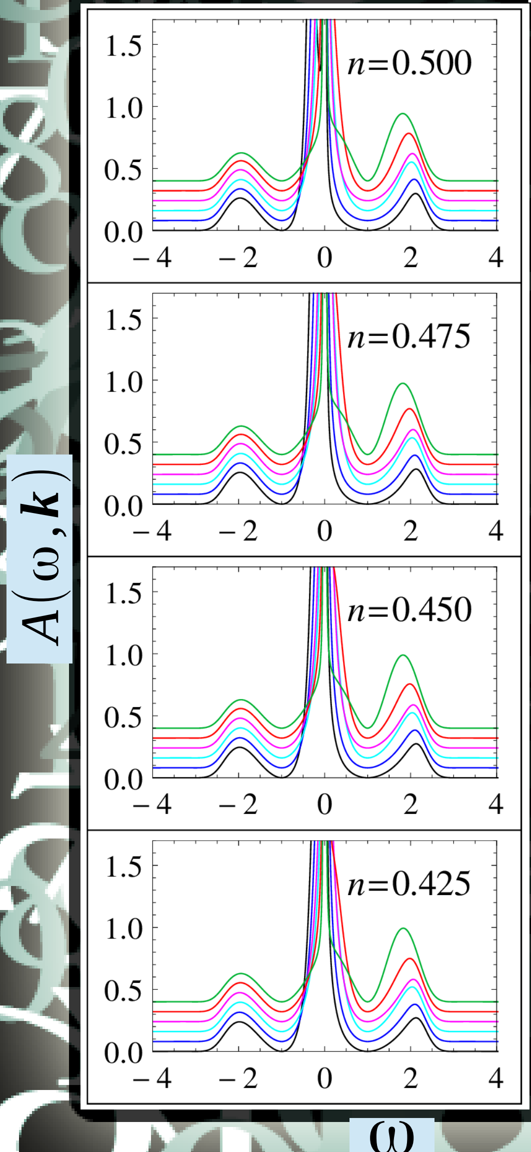
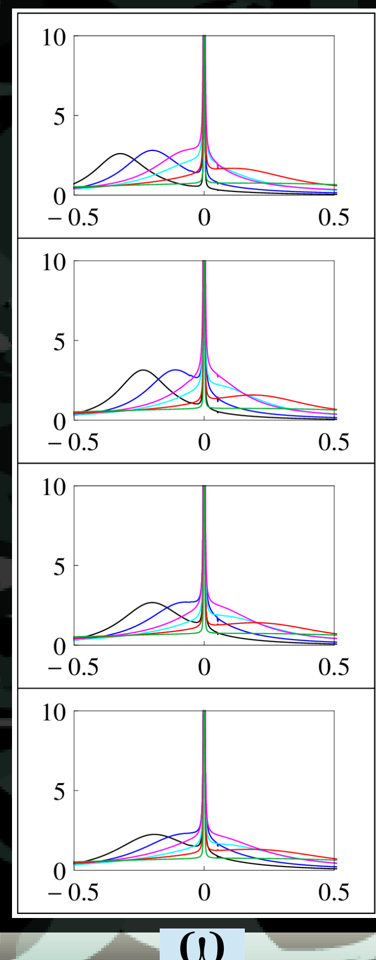
The beauty of holography: real-frequency spectra @low T



Zoom-in at small ω



$T = 0.05$

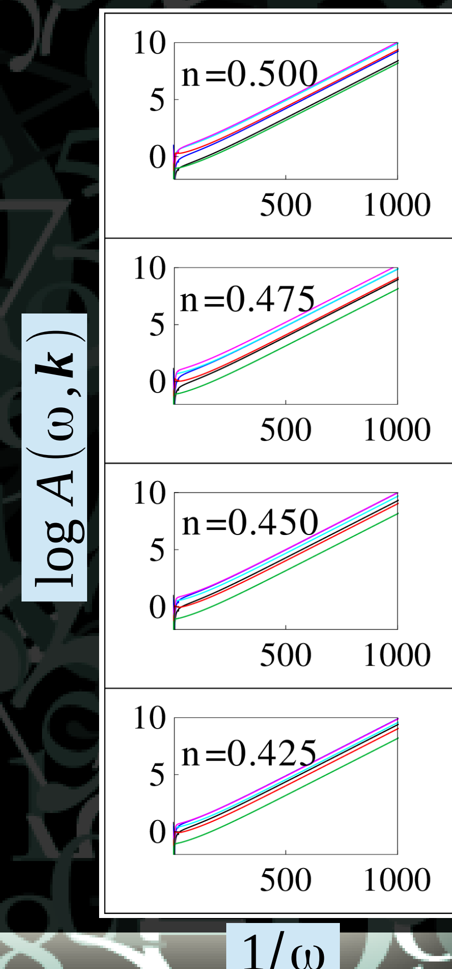
Key points:

1) very sharp quasiparticle at low temperature

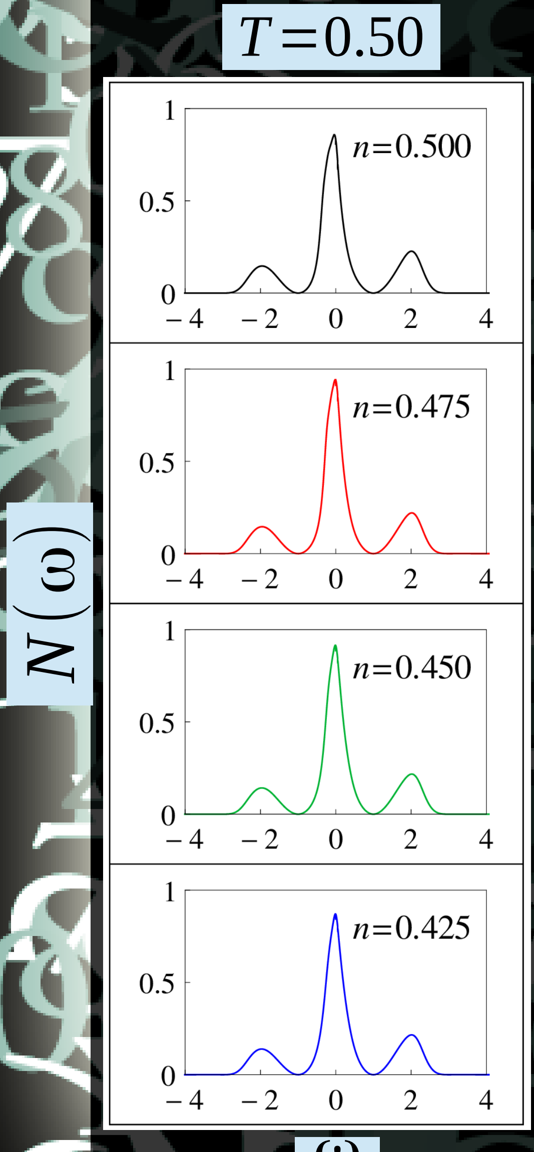
2) exponentially narrow peaks $\log A(\omega) \propto 1/(\omega + a_0)$

3) compare to Iizuka et al self-energies

Zoom-in linearizedl



Local spectral function

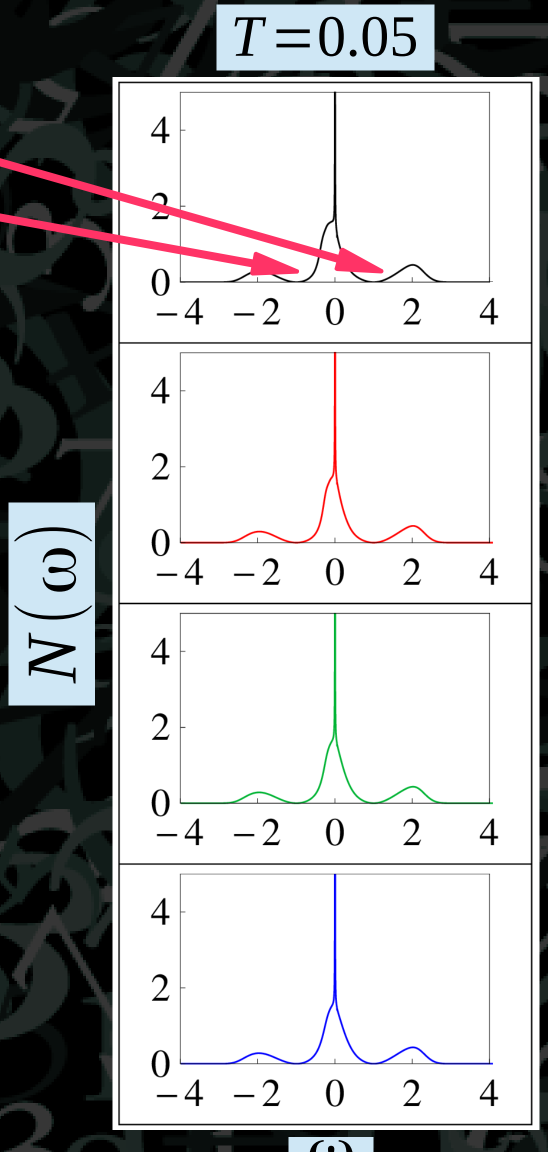


Three-peak structure

Clear separation between the low-energy and high-energy spectrum

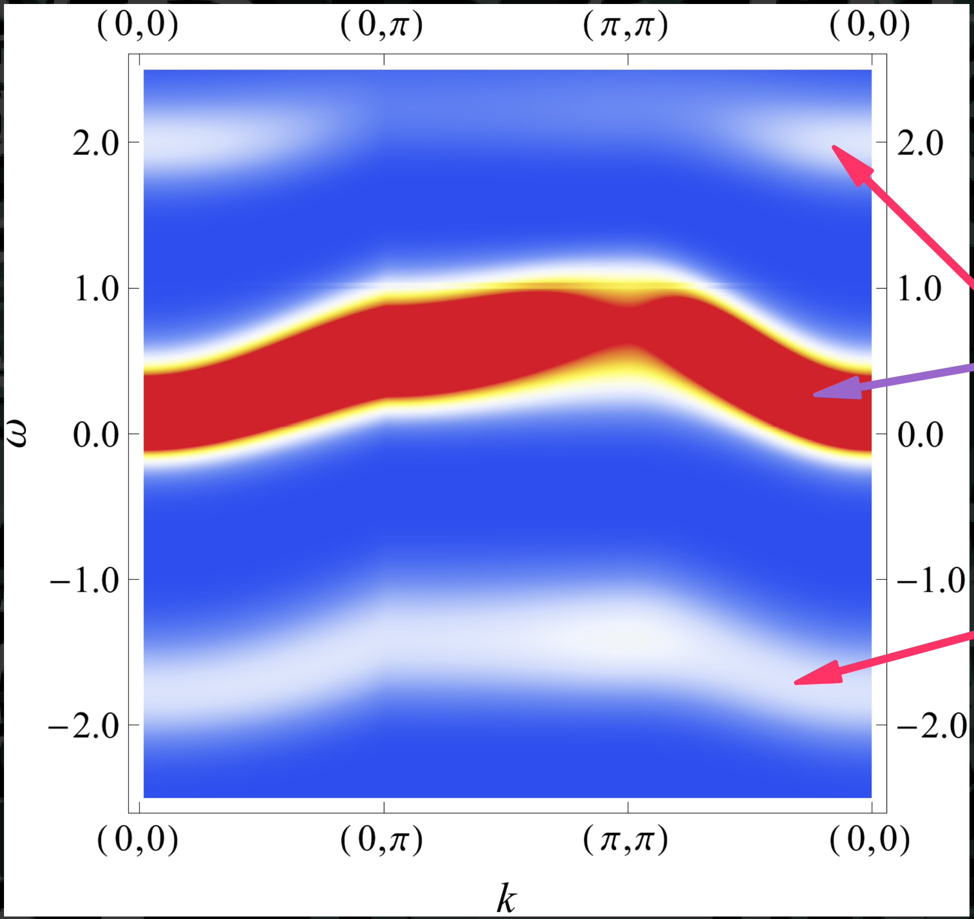
A signature of Mottness?

The puzzle: no sign of gap at half-filling. Strange but not totally inconsistent with QMC data



Spectral map in (ω,**k**) plane

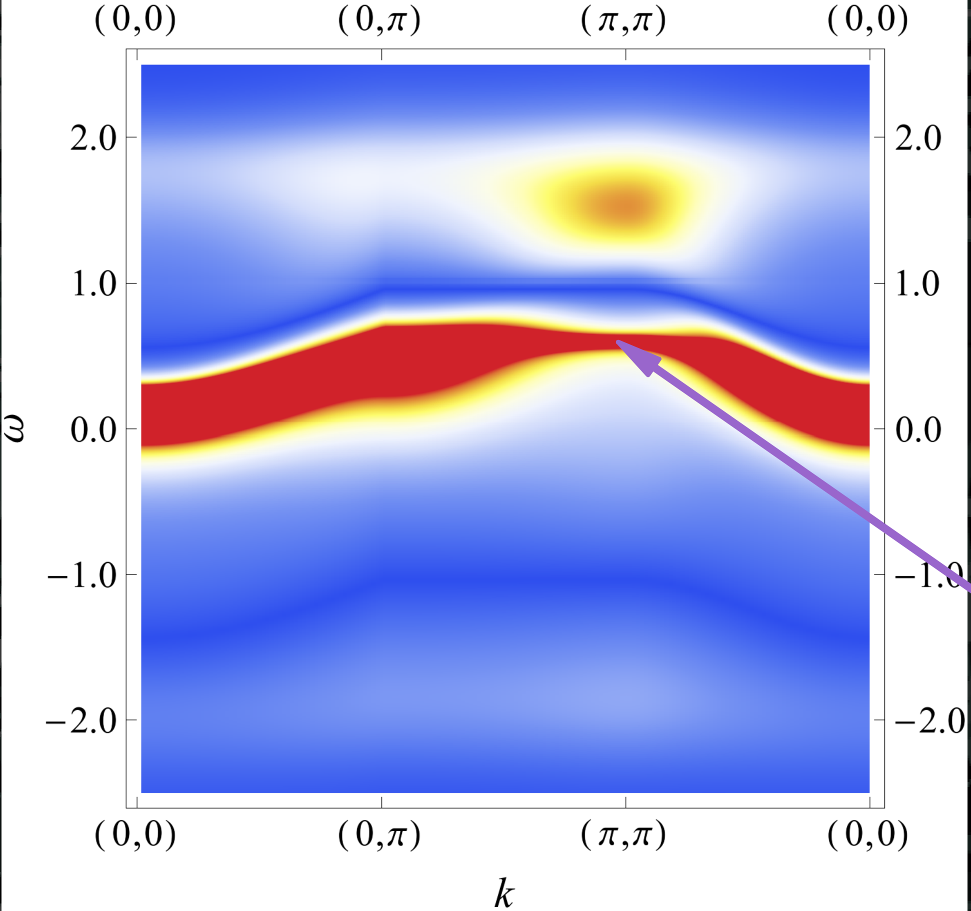
$T = 0.50, \quad n = 0.425 \Rightarrow \mu = 2.50$



Broad peak

Hubbard bands

The beauty of holography: spectral map in (ω,**k**) plane @low T



$T=0.05, \quad n=0.425 \Rightarrow \mu=2.50$

Hubbard bands plus a very sharp quasiparticle

Something is happening at $\boldsymbol{k}=(\pi,\pi)$

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Holography lite 1: semiholographic lattice

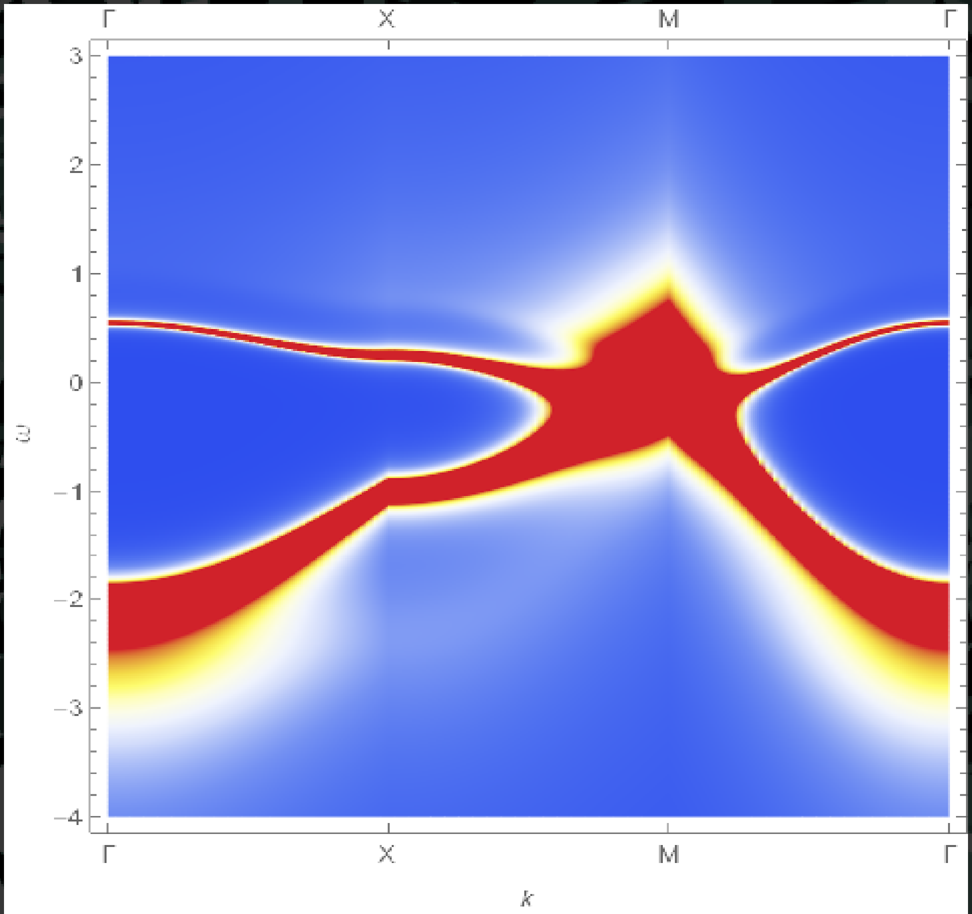
- The idea by Polchinski and Faulkner 2011, applied in several contexts so far, e.g. recent study of momentum-dependent scaling exponents [2112.06576]
- Couple the holographic propagator in the plane to a free lattice fermion:

$$\widetilde{G}_R(\omega,\boldsymbol{k})=\left(\omega-\cos\left(ak_x\right)-\cos\left(ak_y\right)-G_R(\omega,k)\right)^{-1}$$

- Uncontrolled but should capture the IR behavior

Holography lite 1: semiholographic lattice

- The simplest possible model – no anisotropy at all in AdS calculations

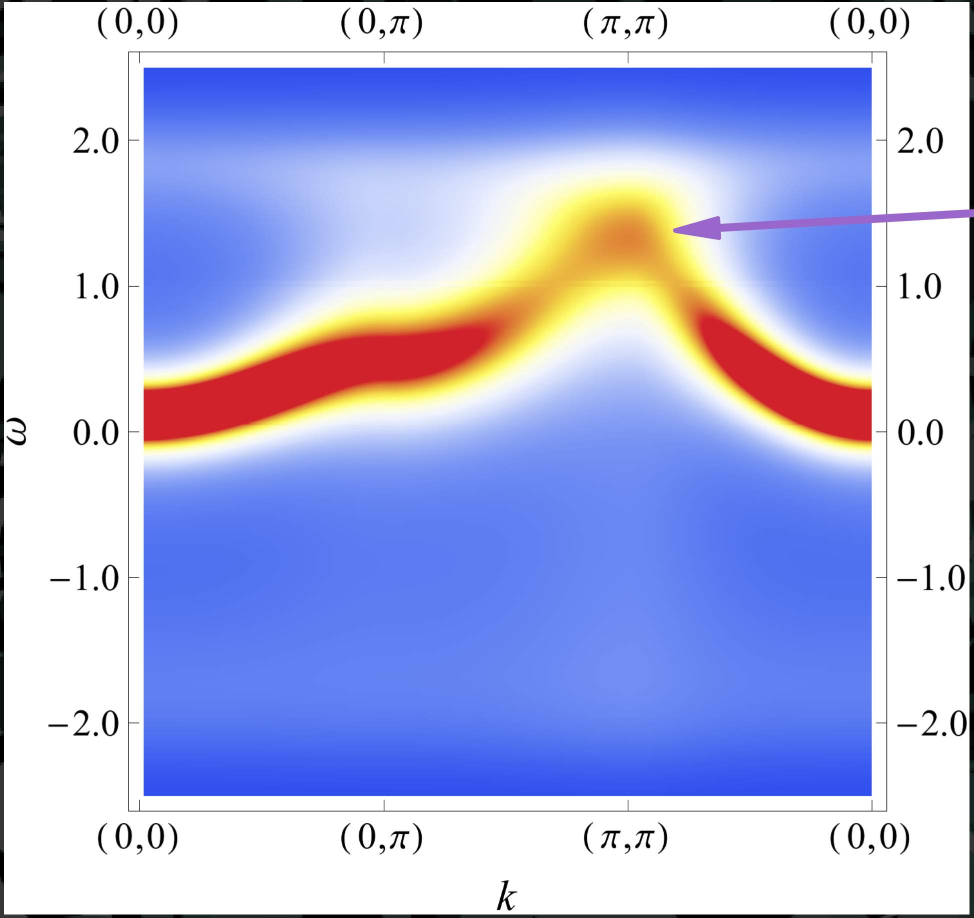


Quasiparticle present but no clearly separated bands

But this is mainly from the lack of dipole coupling!

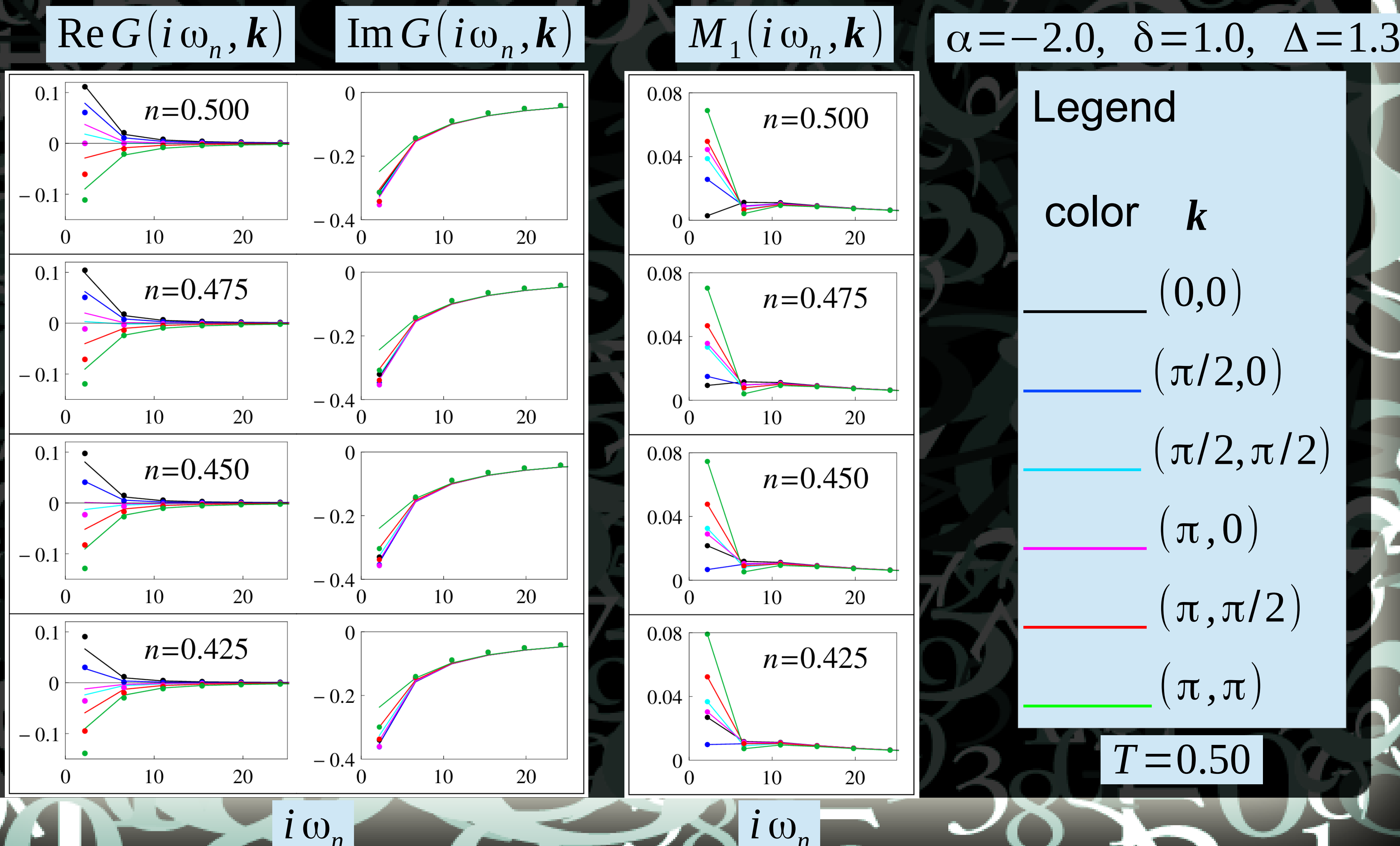
Holography lite 2: multipole expansion

- Holographic lattice but avoid solving the PDEs
- Expand the stress tensor and separate the variables

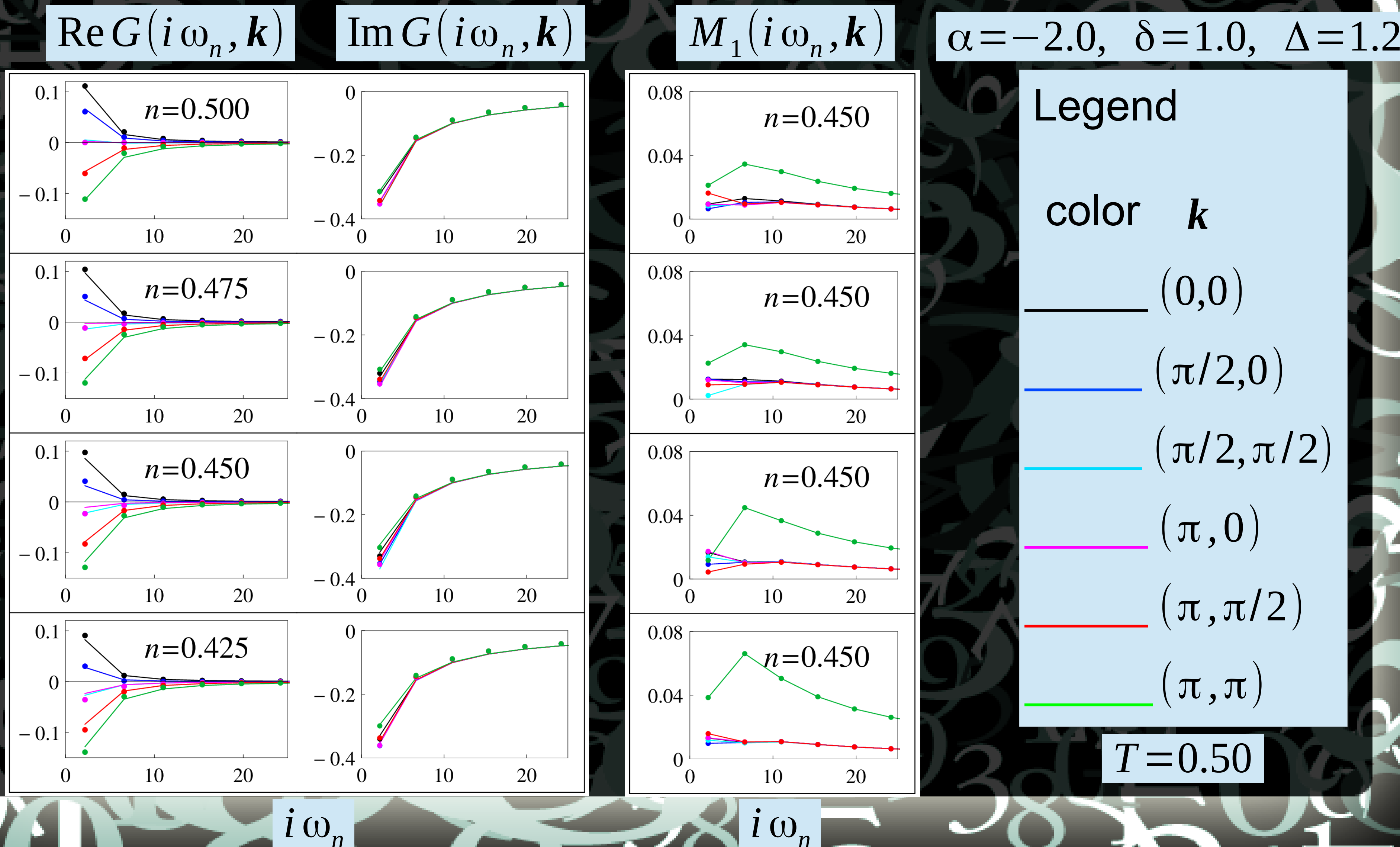


No dipole coupling ⇒ Quasiparticle almost disappears around $\boldsymbol{k}=(\pi,\pi)$

Holography lite 2: multipole expansion - NO dipole coupling



Holography lite 2: multipole expansion WITH dipole coupling



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How seriously are we to take this?

- Matsubara EDCs of CTINT Hubbard model well-described by holographic EDCs in EMD background
- Real-frequency spectra overall look as expected but a few surprises are there: no gap even @low T at half-filling, pecular behavior for $\boldsymbol{k}=(\pi,\pi)$

How seriously are we to take this?

- ***Just a good fit or something deeper?***
- ***Is it time for holographers to tackle microscopic models in a controlled way?***

How seriously are we to take this?

- ***Just a good fit or something deeper?***
- ***Is it time for holographers to tackle microscopic models in a controlled way?***
- <u>To do's:</u>
- IR analysis a la Hong Liu (on the lattice) – can we say something about the T=0 behavior of the Hubbard model?
- Strong lattice regime (work in progress by Filip&Vladan)
- Transport properties & relation to experiment