Quantum supreme matter and the strange metals.

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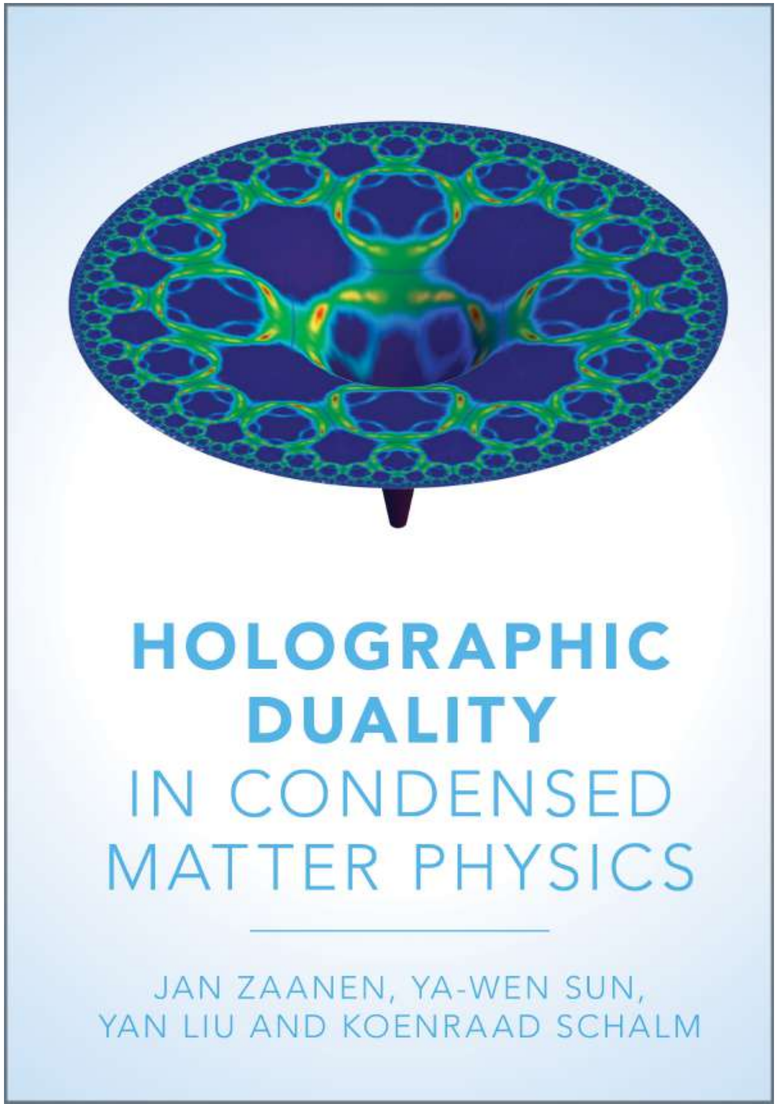
Quantum computing: information/entanglement

String theory/qu. gravity: the AdS/CFT correspondence

!?!

Experiment: condensed matter, high Tc etc.

State of the art 2015



Solliciting feedback, arXiv:2110.00961

Lectures on quantum supreme matter.

Jan Zaanen

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(Dated: December 28, 2021)

Abstract

These notes are based on lectures serving the advanced graduate education of the Delta Institute of Theoretical Physics in the Netherlands in autumn 2021. The goal is to explain in a language that can be understood by non-specialists very recent advances in quantum information and especially string theory suggesting the existence of entirely new forms of matter. These are metallic states characterized by an extremely dense many body entanglement, requiring the supremacy of the quantum computer to be completely enumerated. The holographic duality discovered in string theory appears to be a mathematical machinery capable of computing observable properties of such matter, suggesting the presence of universal general principles governing its phenomenology. The case is developing that these principles may well apply to the highly mysterious physical properties observed in the high temperature superconductors and other strongly interacting electron systems of condensed matter physics.

The relations with quantum gravity

arXiv:2205.02285

Statistical mechanics of strange metals and black holes

Subir Sachdev

Department of Physics, Harvard University, Cambridge MA-02138, USA

School of Natural Sciences, Institute for Advanced Study, Princeton, NJ-08540, USA and

International Centre for Theoretical Sciences,

Tata Institute of Fundamental Research, Bengaluru 560 089, India

(Dated: May 6, 2022)

Abstract

A colloquium style review of the connections between the Sachdev-Ye-Kitaev model and strange metals without quasiparticles, and between the SYK model and the quantum properties of black holes.

To appear in *ICTS News*,

newsletter of the International Centre for Theoretical Sciences, Tata Institute of Fundamental Research.

How it started …

analysis of TDH transcripts expressed in human fetal liver tissue showed complete skipping of exon 4 and either complete skipping or aberrant splicing of exon 6 (fig. S8). Given that exons 4 and 6 encode segments of the enzyme critical to its function and that truncation via the nonsense codon at amino acid 214 would also be predicted to yield an inactive variant, it appears that the human gene is incapable of producing an active TDH enzyme. Remarkably, all metazoans whose genomes have been sequenced to date, including chimpanzees, appear to contain an intact TDH gene (*14*). Unless humans evolved adaptive capabilities sufficient to overcome three mutational lesions, it would appear they are TDH deficient.

Human ES cells grow at a far slower rate than mouse ES cells, with a doubling time of 35 hours (*15*). Whether the slower growth rate of human ES cells reflects the absence of a functional TDH enzyme can perhaps be tested by introducing, into human ES cells, either a repaired human TDH gene or the intact TDH gene of a closely related mammal. That this strategy might work is supported by the expression in human cells of a functional form of the 2-amino-3-ketobutyrate-

String Theory, Quantum Phase Transitions, and the Emergent Fermi Liquid

Mihailo Čubrović, Jan Zaanen, Koenraad Schalm*

A central problem in quantum condensed matter physics is the critical theory governing the zero-temperature quantum phase transition between strongly renormalized Fermi liquids as found in heavy fermion intermetallics and possibly in high–critical temperature superconductors. We found that the mathematics of string theory is capable of describing such fermionic quantum critical states. Using the anti–de Sitter/conformal field theory correspondence to relate fermionic quantum critical fields to a gravitational problem, we computed the spectral functions of fermions in the field theory. By increasing the fermion density away from the relativistic quantum critical point, a state emerges with all the features of the Fermi liquid.

Quantum many-particle physics lacks a general mathematical theory to deal with fermions at finite density. This is known as the "fermion sign problem": There is no recourse to brute-force lattice models because the statistical path-integral methods that work for any bosonic quantum field theory do not work for finite-density Fermi systems.

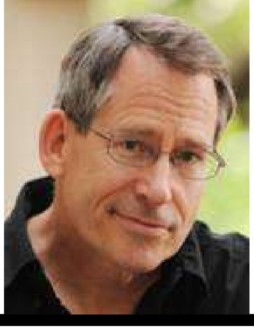
Overview: lessons of holography.

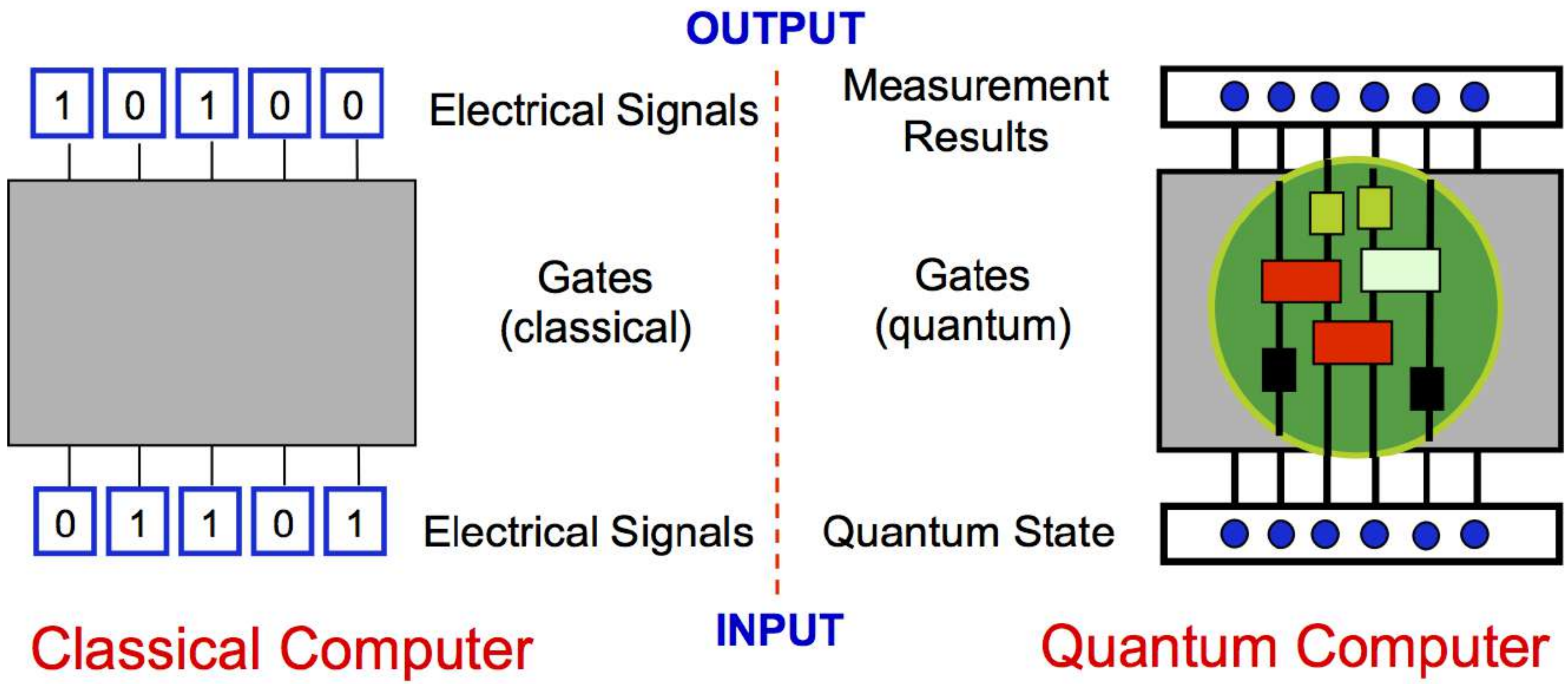
1. Generalizing universality: from scale invariant (stoquastic) to scale covariant (non-stoquastic) RG flow at zero temperature.

2. Finite temperature: bulk black holes encoding for the "Planckian" quantum thermalization.

3. Don't trust the holographic oracle! Fermions and the large N UV sensitivity …

The supremacy of the quantum computer.





Unitary evolution: sequential one- and two bit operations.

Read out: collapsing the wave function, information is processed

Many body/bit Hilbert space.

Two qubits: Hilbert space dimension $2^2 = 4$

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

Three qubits: Hilbert space dimension $2^3 = 8$

$$|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle,$$

Physical world 10^{23} "qubits": Hilbert space dimension $2^{10^{23}}$

$$|\Psi_n\rangle = \sum_{config.i} C_i^n |config.i\rangle$$

Overwhelming amount of quantum information.

Semiclassics: "Short range entangled product states".

Perturbation theory = the textbook diagrams:

$$|\Psi\rangle_0 = A|\Psi\rangle_\otimes + \sum_i a_i |config, i\rangle$$

Computed "around" the classical product state (polynomial complexity).

As long as A is finite: entanglement up to a microscopic length scale ("short range").

On the macroscopic scale: no entanglement info, system is described by classical "field" theory.

Equivalently, "semiclassics": requantize the classical theory.

Conventional condensed matter- and high energy physics = "particle physics"

"The classical condensates: from crystals to Fermi-liquids."

States of matter that we understand are short ranged entangled product!

$$\left|\Psi_0\{\Omega_i\}\right\rangle = \Pi_i \hat{X}_i^+(\Omega_i)\left|vac\right\rangle$$

- Crystals: put atoms in real space wave packets $X_i^+(R_i^0) \propto e^{(R_i^0 - r)^2/\sigma^2}\psi^+(r)$

- Magnets: put spins in generalized coherent state

$$X_i^+(\vec{n}_i) \propto e^{i\varphi_i/2}\cos(\theta_i/2)c_{i\uparrow}^+ + e^{-i\varphi_i/2}\sin(\theta_i/2)c_{i\downarrow}^+$$

- Superconductors/superfluids: put bosons/Cooper pairs in coherent superposition

$$X_{k/i}^+ \propto u_k + v_k c_{k\uparrow}^+ c_{-k\downarrow}^+, \quad u_i + v_i e^{i\varphi_i} b_i^+$$

- Fermi gas/liquid: product state in momentum space (mod Pauli principle).

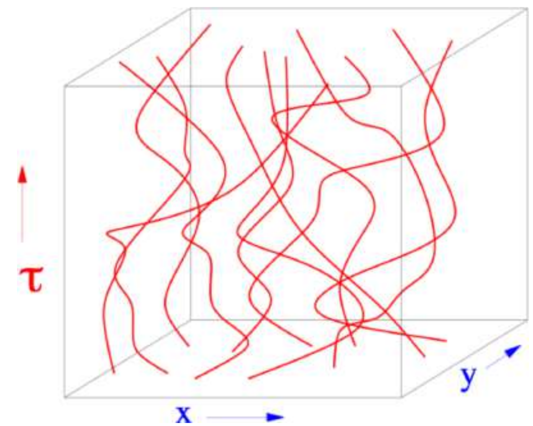
$$\left|\Psi_{FL}\right\rangle = \Pi_k^{k_F} c_k^+\left|vac\right\rangle$$

Fermions at a finite density: the sign problem.

Imaginary time first quantized path-integral formulation

$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta)$$

$$= \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

- negative Boltzmann weights
- **in general NP-hard problem (Troyer, Wiese)!!!**

“Quantum supreme matter”

Do non-SRE states of matter exist, where many body entanglement governs the physical properties?

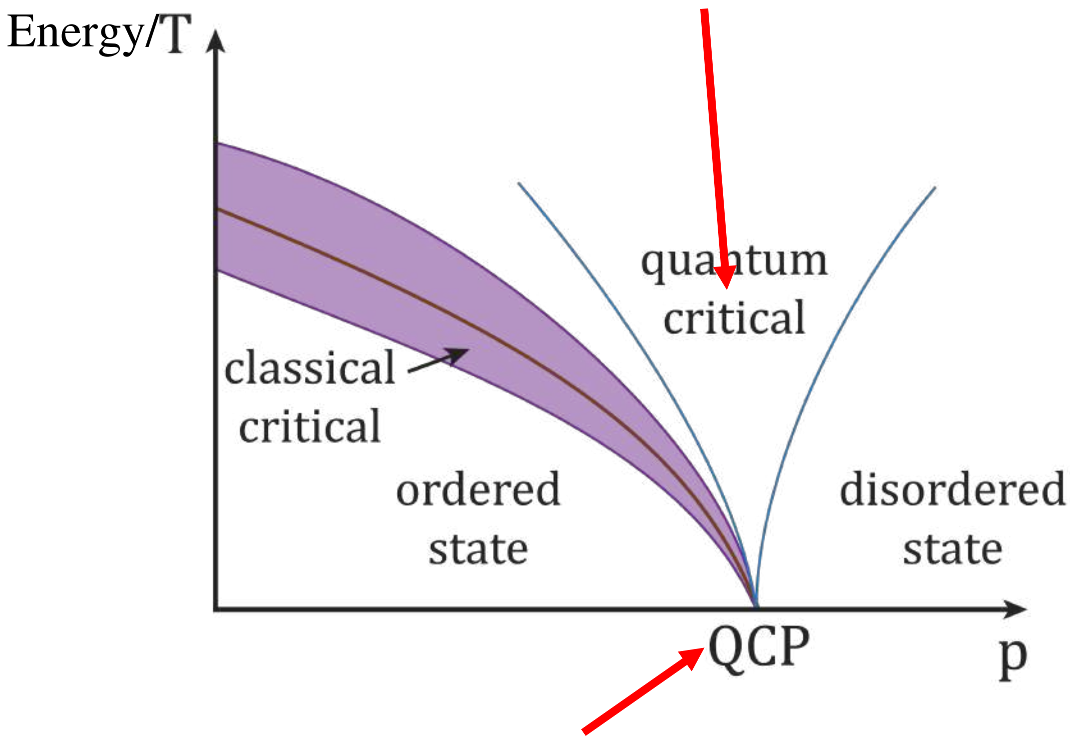
$$|\Psi_0\rangle = \sum_{i=1}^{2^N} C_i^0 \, |config, i\rangle$$

1. **Incompressible matter** (X.-G. Wen, 1992): macroscopic entanglement underneath the topological order described by topological field theory (frac. quantum Hall, etc). The ***entanglement is extremely sparse***, infinite number of microscopic qubits needed for a topological quantum bit.

2. Compressible matter: by default very densely entangled, little is known. Since the 1990’s strongly interacting CFT’s/”Sachdevean” quantum criticality. Recent developments: SYK and the big ***AdS/CFT*** machine**.**

The stoquastic view: "conventional quantum criticality".

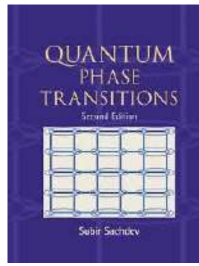
Scale invariance is dynamically generated "inside the wedge" anchored at …



… an isolated point in coupling constant space.

“Sign free” = “stoquastic” quantum criticality.

Sudip Chakravarty
“CHN”, 1988



Subir Sachdev,
1990’s

String theorists: they found it mathematically obvious…

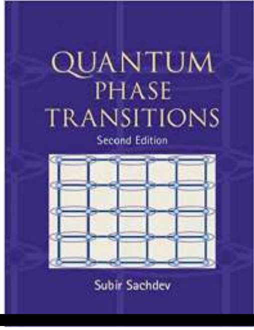
1. Write down problem in path integral.

2. “Wick rotate” from Lorentzian to Euclidean signature (“imaginary time”).

3. No signs (“stoquastic”): the path integral is identical to the (stochastic) partition sum of an equivalent statistical physics problem in d+1 dimensions.

4. This gets cool dealing with a quantum phase transition.

Strongly interacting "stoquastic" quantum critical states.



$$S = \int d^d x d\tau \left[(\partial_\tau \Phi)^2 + (\nabla \Phi)^2 + m^2 \Phi^2 + w \Phi^4 \right]$$

$D = d + z < D_{u.c.} (= 4) : \quad w \neq 0$ at the IR fixed point

"strongly interacting" = NP-hard (critical slowing down in QMC) = densely entangled quantum critical state.

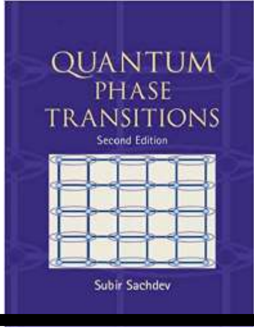
$$\langle \Phi \Phi \rangle \sim \frac{1}{\sqrt{k^2 - \omega^2}^{2-\eta}}$$

$D \geq D_{u.c.} \quad w = 0$ at the IR fixed point

Mean-field fixed point: SRE product state characterized by particles in its spectrum:

$$\langle \Phi \Phi \rangle \sim \frac{1}{k^2 - \omega^2}$$

Stoquastic universality.



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$D = d + z < D_{u.c.} (= 4) : \quad w \neq 0$ at the IR fixed point

Universality rules: the exponential complexity of the states renders a "perfect averaging" of the VEV's. Symmetry and dimensionality determine the scaling dimensions of the marginal operators. Irrelevant operators "disappear".

$D \geq D_{u.c.} \quad w = 0$ at the IR fixed point

Mean-field fixed point: away from the fixed point a multitude of (perturbative) irrelevant operators switches on, universality is destroyed. "Above 4 dimensions water-steam and Ising are not the same."

AdS-CFT and zero density unparticle physics.

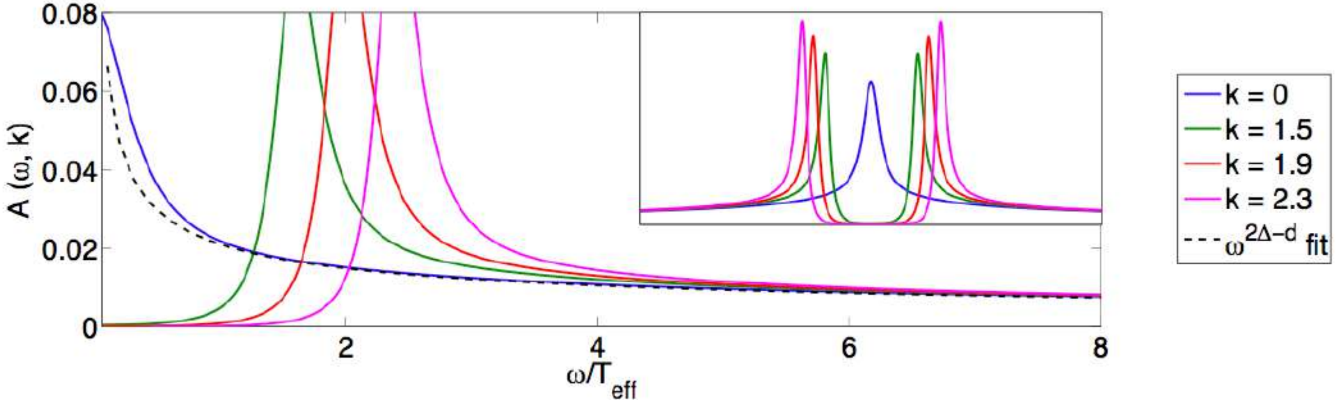

Schalm Cubrovic

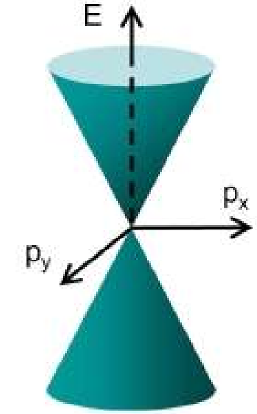
Fermion spectral functions of quantum critical relativistic ("Dirac") fermions at small but finite temperature = Dirac quantum mechanics in the AdS-Schwarzschild bulk.

Science 325, 439 (2009)

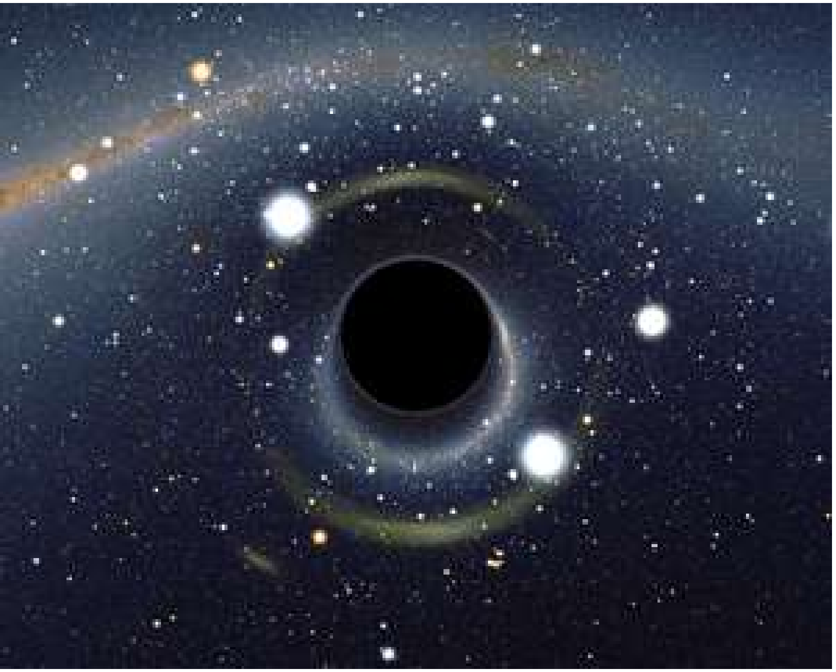
$$\langle O(\vec{q},\omega)O(0)\rangle = \frac{1}{(q^2-\omega^2/c^2)^{D-\Delta}}$$

Spectral weight has to disappear outside the light cone.





Black holes as "maximal entanglement" quantum computers" !?



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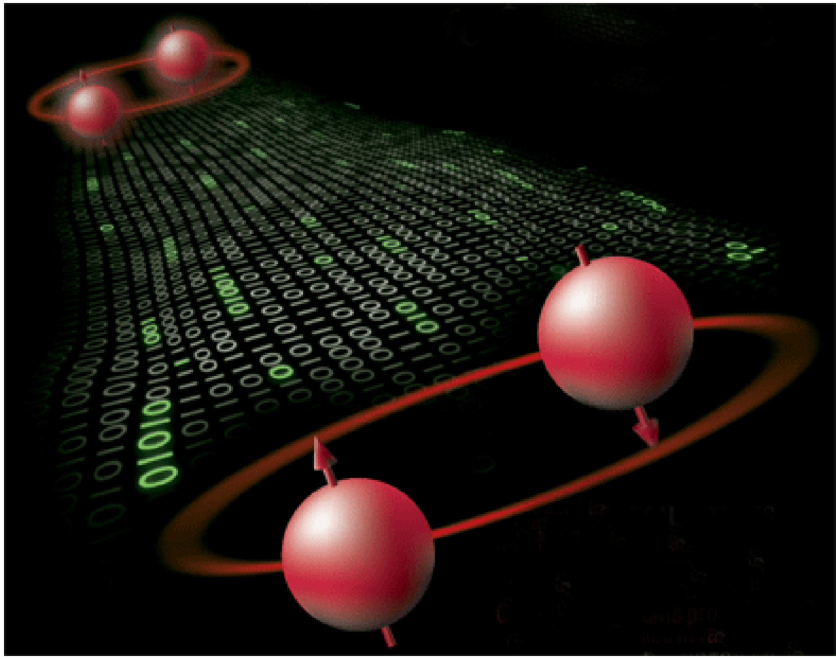
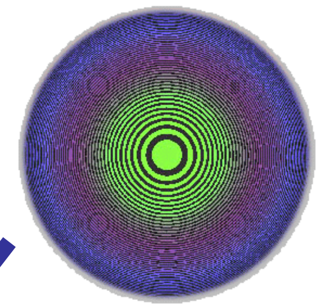


Holographic gauge-gravity duality

Einstein Universe "AdS"

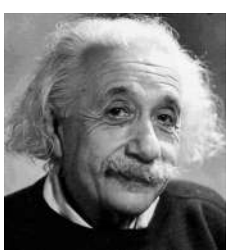
't Hooft-Susskind holographic principle

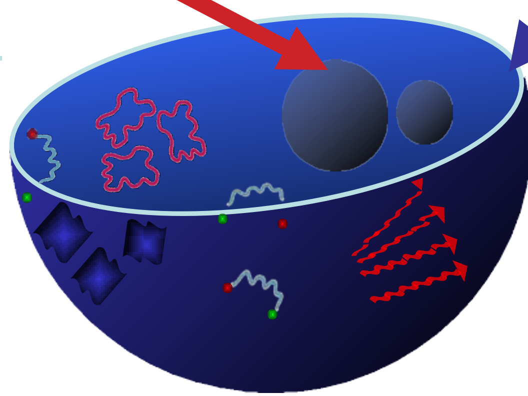
Quantum field world "CFT"



Classical general relativity

Uniqueness of GR solutions



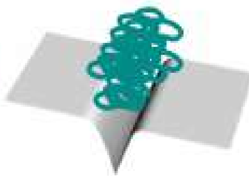


Extremely strongly entangled quantum matter

"Generating functional of matter emergence principle"

AdS/CFT correspondence

Closed string description



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Open string description

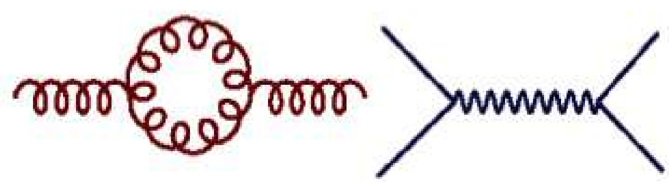
Low energy limit with large N

Supergravity on $AdS_5 \times S^5$

dual

D=4 $\mathcal{N}$=4 U(N) Super Yang-Mills

×



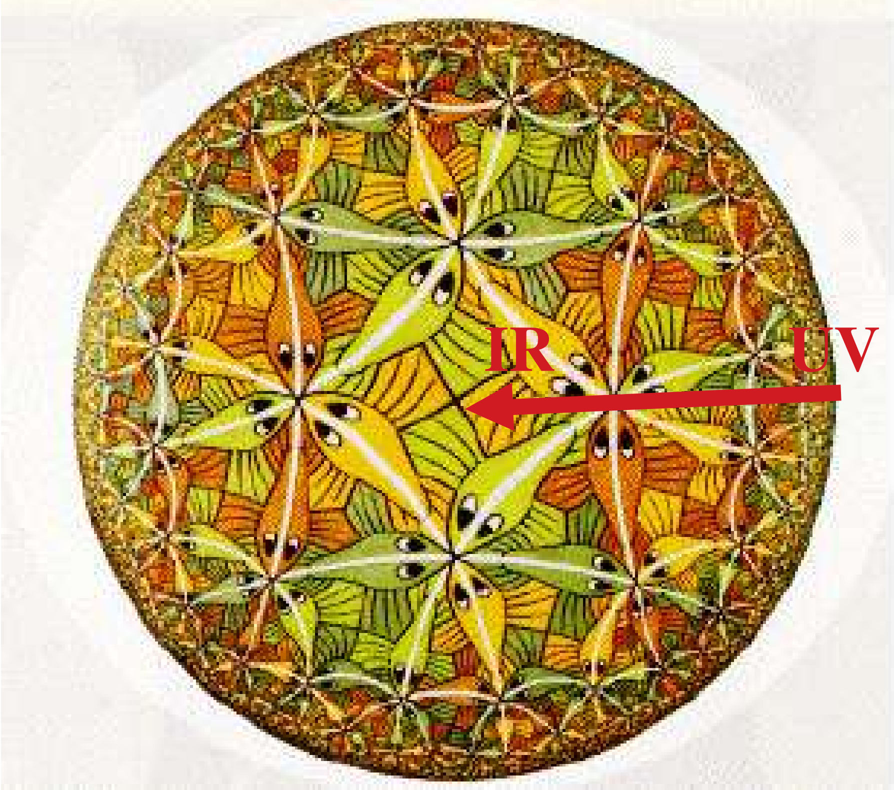
Good approximation for strong coupling

$$\lambda = g_{\rm YM}^2 N \gg 1$$

Good approximation for weak coupling

$$\lambda = g_{\rm YM}^2 N \ll 1$$

General Relativity = Renormalization Group



Extra radial dimension of the bulk <=> **scaling "dimension" in the field theory**

Bulk AdS geometry = **scale invariance of the field theory**

Originating in a deep geometrical principle: the *isometry* of a *curved* space in D+1 dimensions is equivalent to *symmetry* in a D dimensional flat manifold.

Bulk isometry and the symmetry of the boundary RG flow.

The strongly interacting stoquastic (zero density) quantum critical state is controlled by conformal invariance (”CFT”) insisting on a perfect scale invariance of the physics.

The RG flow is “geometrized” in the bulk, where the extra “radial dimension” is associated with the scaling direction.

This is encoded in the curved geometry of the bulk and Anti-de-Sitter is unique with the property of scale invariance of its metric

$$ds^2_{\mathrm{AdS}} = \frac{1}{r^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu + dr^2\right)$$

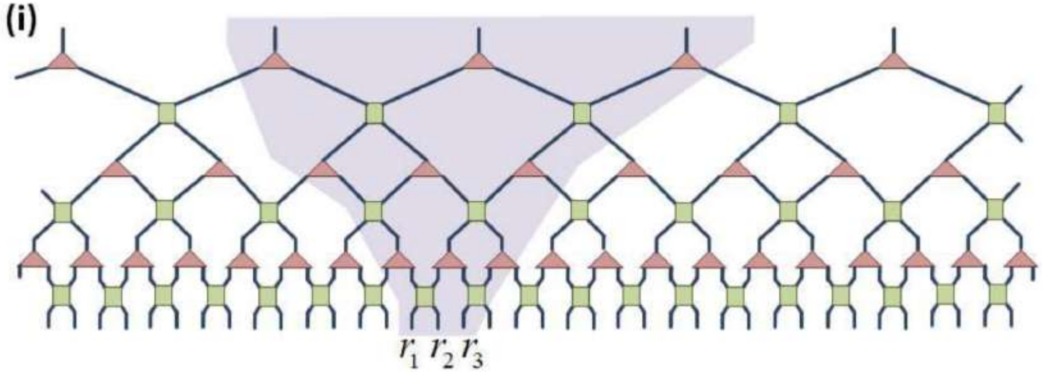
$$x^\mu \to \Lambda x^\mu, r \to \Lambda r: \; ds^2_{\mathrm{AdS}} \to ds^2_{\mathrm{AdS}}$$

Finite temperature interrupts this but its effect is impeccably captured by the Schwarzschild black hole metric in the deep interior.

Central charge and quantum complexity of CFT's

Vidal

Hierarchical tensor networks (MERA): pragmatic approximation scheme "stitching together" the vacuum entanglement CFT's in terms of local tensors. For a non-integrable CFT this becomes exact only when the bond dimension of the local tensors becomes "2^N".

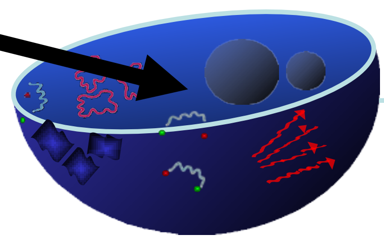


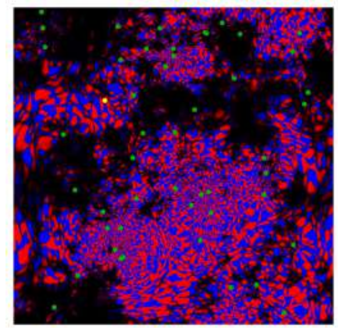
Evenbly and Vidal (arXiv:1109.5334): to maintain the same accuracy the bond dimension has to increase by e^c with the central charge $c = N^2$. In this sense the large N limit CFT's are maximally entangled.

The charged back hole encoding for finite density (2008 - ????)

Anti de Sitter universe.

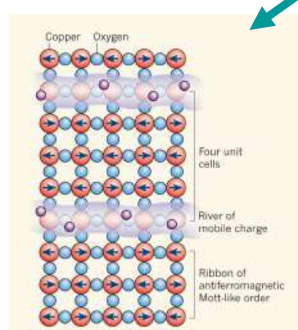
Finite density quantum matter:





Holographic strange metals

Charged black hole in the middle



Stripy pseudogap orders

High Tc superconductors

Emergent Fermi liquids

"Scaling atlas" of holo strange metals

Gouteraux

Kiritsis

Deep interior geometry sets the scaling behavior in the emergent deep infrared of the field theory. Uniqueness of GR solutions:

1. "Cap-off geometry" = confinement: conventional superconductors, Fermi liquids ….

2. Geometry survives: "hyperscaling violating geometries" (Einstein – Maxwell – Dilaton – Scalar fields – Fermions).

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds$$

$$S \mu T^{(d-\theta)/z}$$

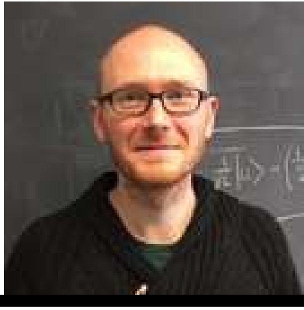
Quantum critical phases with unusual values of:

$z =$ **Dynamical critical exponent**

$\theta =$ **Hyperscaling violation exponent**

$\zeta =$ **Charge exponent**

Finite fermion density: the degeneracy scale.



e.g., Phys. Rev. D 100, 086020 (2019)

Fermion signs impose a dense "nodal structure" on the total wave function.

This implies the presence of a large zero-point motion energy: the Fermi energy in a Fermi-liquid.

How to accommodate this fundamental scale in a scale "invariant" quantum supreme generalization?

Holographic "RG = GR" answer:

Instead of the *scale invariance* of the geometrized RG flow of "stoquastic" quantum criticality, at finite density it is controlled by *covariance under scale transformation*!

The isometry of RG flow.

Stoquastic criticality RG flow controlled by *scale invariance of the bulk metric* (including finite T, etc.):

$$ds^2_{\mathrm{AdS}} = \frac{1}{r^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu + dr^2\right)$$

$$x^\mu \to \Lambda x^\mu, r \to \Lambda r: \; ds^2_{\mathrm{AdS}} \to ds^2_{\mathrm{AdS}}$$

Finite density strange metal RG flow controlled by *scale covariance of the bulk metric*:

$$ds^2_{\mathrm{EMD}} = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2\right)$$

$$x^\mu \to \Lambda x^\mu, r \to \Lambda r: \; ds^2_{\mathrm{EMD}} \to \Lambda^{2\theta/d} ds^2_{\mathrm{EMD}}$$

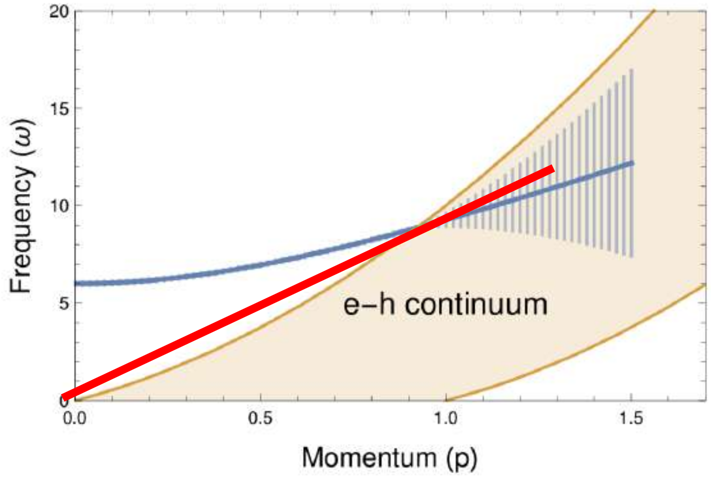
E.g., the degeneracy scale (“E_F”) is remembered.

This reconstructs the Fermi liquid RG flow as a special “engineering scaling dimensions” case!

The density/current excitation spectrum of the Fermi liquid.

Zero sound ($F^0_s > 0$, coherent vibration of the Fermi surface) and Lindhard continuum.

$$\sigma(\omega,p) = \frac{i\omega}{p^2}\Pi(\omega,p) \qquad \chi(\omega,p) = \frac{\Pi(\omega,p)}{1 - V_p\Pi(\omega,p)}$$



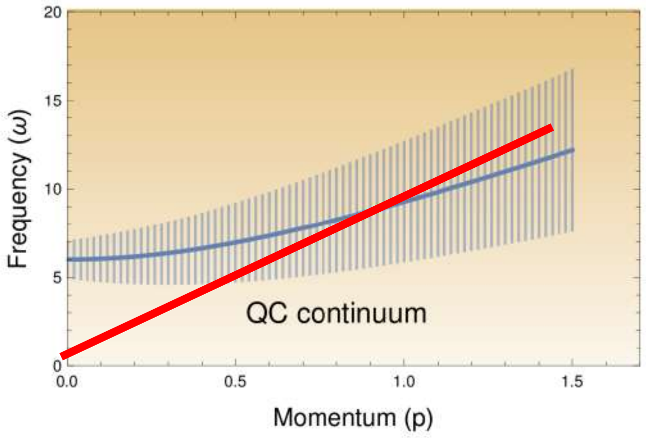
Two parallel sectors:

$$\Pi(\omega,p) = \Pi_S(\omega,p) + \Pi_L(\omega,p)$$

$$\Pi_L \sim p^2\omega => \sigma_L \sim \omega^2$$

The density/current excitation spectrum of the holo strange metals.

The strange metals also carry zero sound but now the "quantum critical" continuum has to obey scaling characterized by anomalous dimensions:

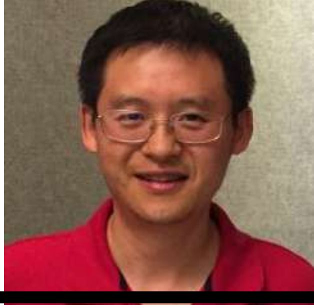


Two parallel sectors: $\Pi(\omega,p) = \Pi_S(\omega,p) + \Pi_{QC}(\omega,p)$

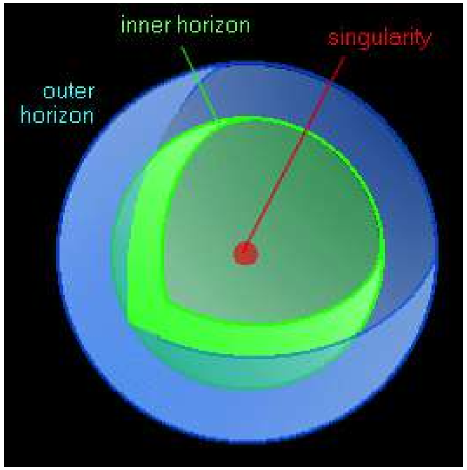
$$\Pi_{QC}(\omega,p) = \omega^{(d-2-\theta+z)/z} F\left(\frac{\omega}{p^z}\right)$$

$$\sigma_{QC}(\omega) \sim \omega^{(d-2-\theta)/z}$$

Finite density: the Reissner-Nordstrom strange metals.



Hong Liu



Near-horizon geometry of the extremal RN black hole:

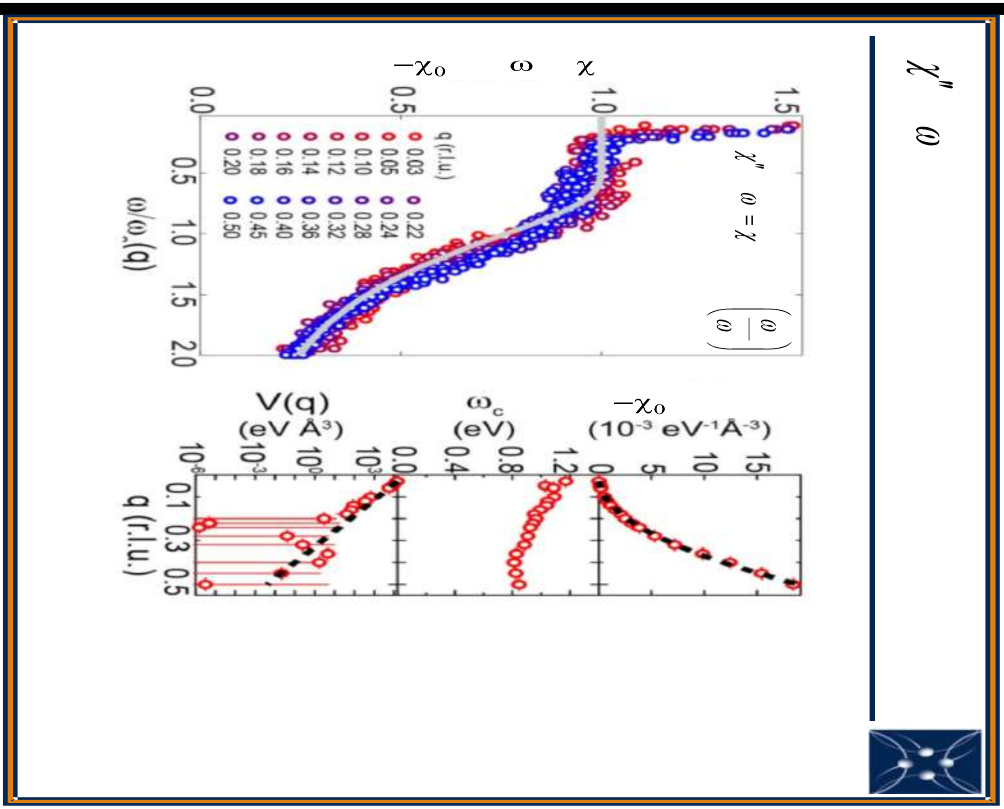
- Space directions: flat, codes for simple Galilean invariance in the boundary.

- Time-radial(=scaling) direction: emergent AdS_2, codes for emergent temporal scale invariance!

Dynamical critical exponent z: $t \sim l^z \quad z \to \infty$

Non-Wilsonian renormalization property: *local quantum criticality.*

The dynamical charge susceptibility (EELS) of the strange metal.



No sign of Lindhard continuum.

This continuum is momentum independent: "local quantum criticality".

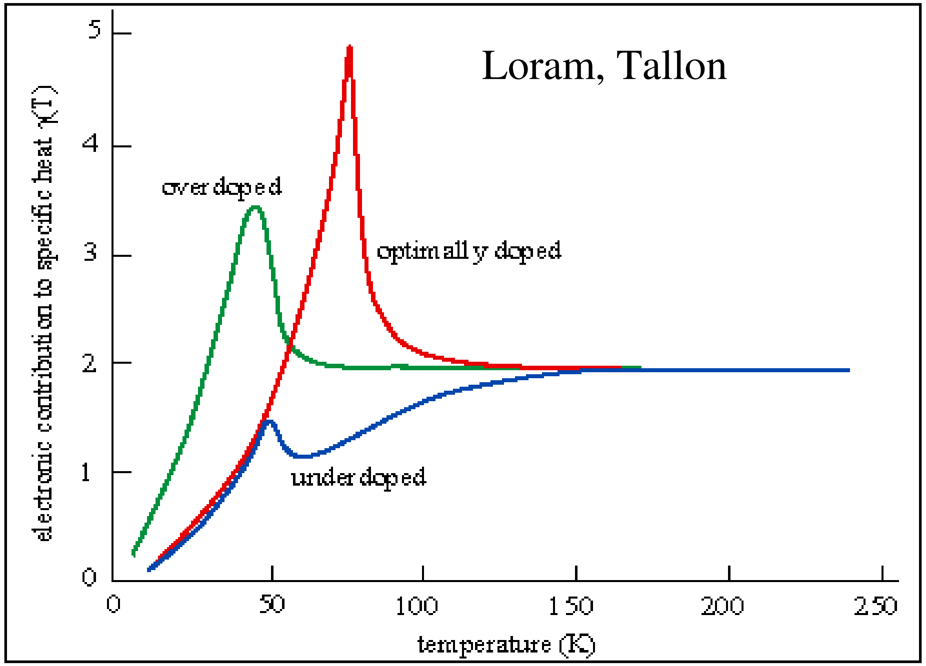
Like a holographic QC scaling continuum: z is obviously infinite!

Energy scaling dimension is zero ("marginal"): different from the "2/3" in the optical conductivity

Abbamonte group: PNAS 201721495 (2018)

Sommerfeld entropy and local quantum criticality.

Loram and Tallon measured the electronic specfic heat in the ”high” temperature strange metal regime, claiming it to be Sommerfeld (“Fermi-liquid”).



$$C = \gamma T \Rightarrow S = T / \mu$$

“Scaling atlas” of holo strange metals

Gouteraux

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$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds$$

$$S \propto T^{(d-\theta)/z}$$

Quantum critical phases with unusual values of:

$z =$ **Dynamical critical exponent**

$\theta =$ **Hyperscaling violation exponent**

$\zeta =$ **Charge exponent**

Generalizing the Fermi liquid: thermodynamics.

In d dimensions the FL is characterized by a **d-1 dimensional manifold of massless excitations** (Fermi surface): $\theta = d - 1$

Every point on the Fermi surface **scales like a CFT_2:** $z = 1$

$$S \sim T^{(d-\theta)/z} = \left(\frac{T}{E_F}\right)$$

Sommerfeld specific heat

Local quantum critical (Abbamonte etc.): $z \to \infty => S \sim T^0$

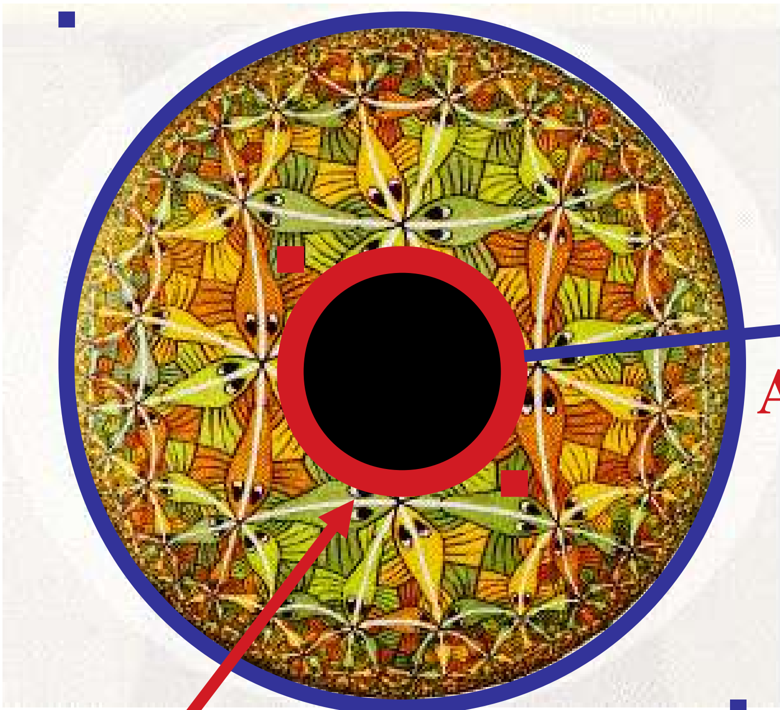
Zero temperature entropy, e.g. Reissner-Nordstrom metal.

Top down "Gubser-Rocha" EMD (conformal to AdS2) metal:

$$-\theta, z \to \infty, \;\; -\theta/z \to 1 \;\; => S = \left(\frac{T}{\mu}\right)$$

The holographic superconductor

Hartnoll, Herzog, Horowitz, arXiv:0803.3295



Condensate (superconductor, …) on the boundary!

AdS-CFT

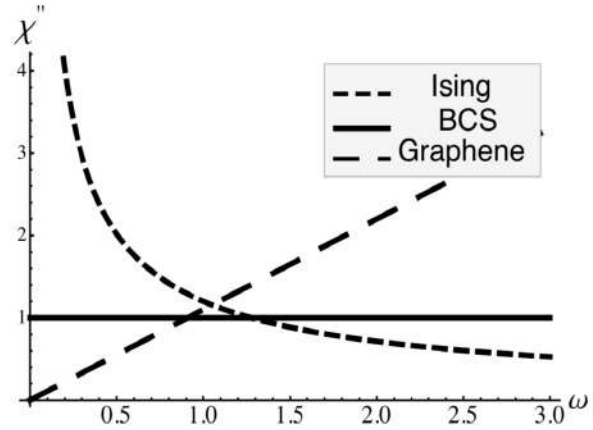
'Super radiance': in the presence of matter the extremal BH is unstable

(Scalar) matter 'atmosphere'

Entanglement, anomalous dimensions and strange metal instability.

" Quantum critical BCS" (J.H. She, JZ, arXiv:0905.1225)

Densely entangled strange metal: the scaling dimension of the pair operator is anomalous.



$$\chi_p(\omega) \sim \frac{1}{\omega^{\alpha_p}}$$

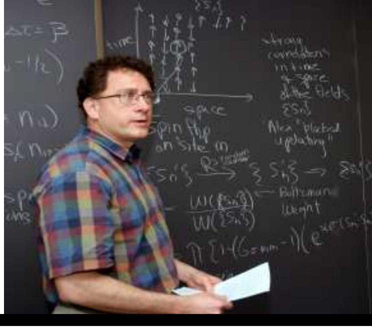
$$1 - g\,\chi'_p(q=0, \omega=0, T=T_c) = 0$$

If relevant $(\alpha_p > 0)$ **Tc is very high even for a weak attractive glue!**

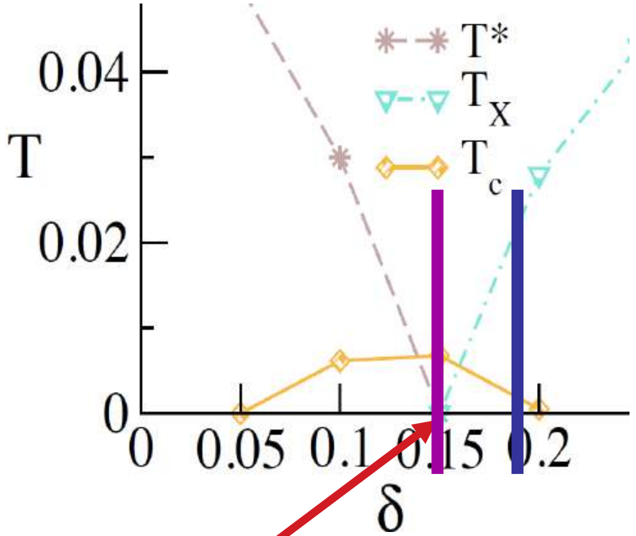
$$=>=> \Delta = 2\omega_B \left(1 + \frac{1}{\lambda}\left(\frac{2\omega_B}{E_F}\right)^{\alpha_p}\right)^{-1/\alpha_p}, \quad \lambda = 2\frac{g}{E_F}\frac{1-\alpha_p}{\alpha_p}$$

Holographic superconductivity works this way! J.H. She et al., arXiv: 1105.5377

Jarrell's DCA quantum criticality (PRL **106**, 047004, 2011)

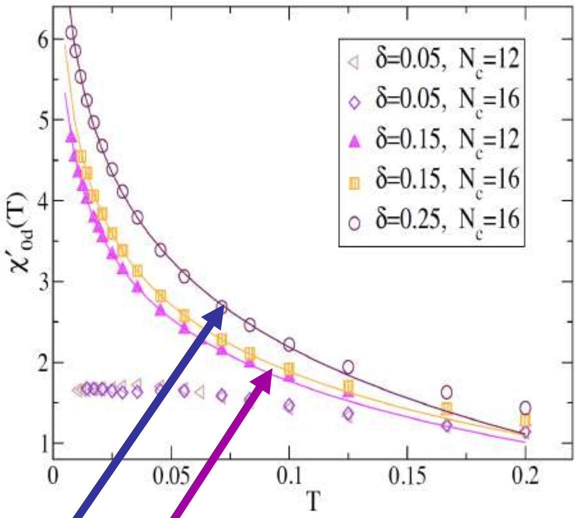


2D Hubbard model:



Phase separation quantum critical end point

Real part "bare" pair susceptibility:



Overdoped: $\chi'_{0d}(\omega = 0) = A \ln(\omega_c/T)$

Optimally doped:

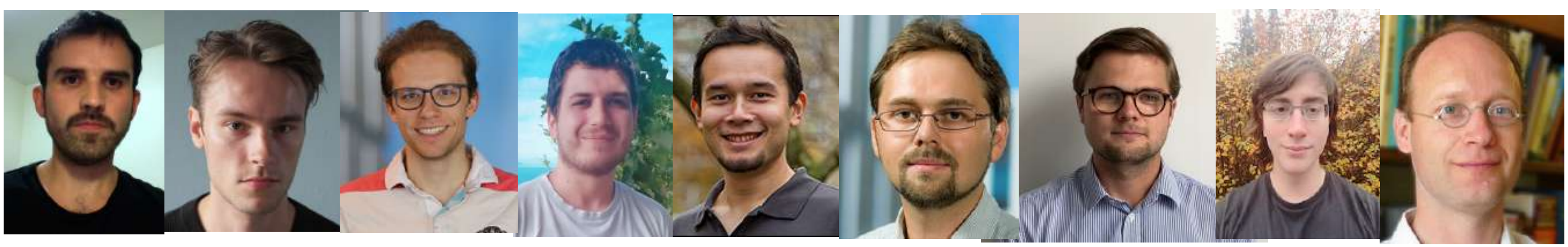
$$\chi'_{0d}(\omega = 0) = B/T^{0.5}$$

The holographic state of the art.

Holography is thoroughly explored for equilibrium in the spatial continuum: the bulk GR is "easy".

Upon breaking translational symmetry (ionic lattices etc.) state of the art numerical GR is required, like black-hole mergers. Little is known!

The Leiden numerical holography effort



Andrade Barcelona | Arend | Balm | Chagnet | Grosvenor | Krikun Nordita | Janse | Rodriguez | Schalm

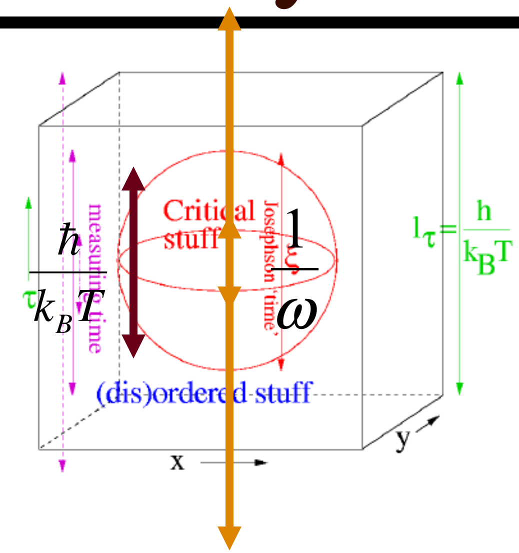
Overview: lessons of holography.

1. Generalizing universality: from scale invariant (stoquastic) to scale covariant (non-stoquastic) RG flow at zero temperature.

2. Finite temperature: bulk black holes encoding for the "Planckian" quantum thermalization.

3. Don't trust the holographic oracle! Fermions and the large N UV sensitivity …

Planckian dissipation and stoquastic quantum criticality



Scaling form dynamical susceptibility:

$$\chi(\omega) \propto \frac{1}{T^{2-\eta}} \Psi\left(\frac{\hbar\omega}{k_B T}\right)$$

Quantum critical regime $\quad k_B T >> \frac{\hbar c}{\xi}$

$$\hbar\omega >> k_B T: \quad \chi(\omega) \propto \frac{e^{i\frac{\pi}{2}(2-\eta)}}{|\omega|^{2-\eta}}$$

$$\hbar\omega << k_B T: \quad \chi(\omega) \propto \frac{1}{T^{2-\eta}} \frac{1}{1 - i\omega\tau_\hbar}$$

Planckian dissipation: $\quad \tau_\hbar = const. \frac{\hbar}{k_B T}, \quad const. = O(1)$

Eigenstate Thermalization hypothesis (Srednicki, Deutscher, 90's)

"Any local observer gets overwhelmed by the enormous amount of quantum info in the physical world to such an extent that he/she does not know better than that everything becomes a thermal state at long times."

Precise formulation:

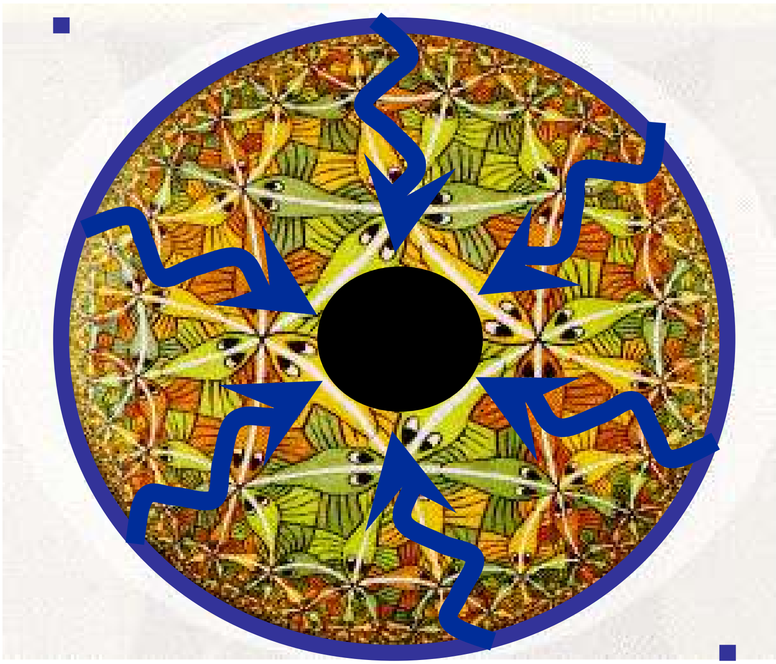
$$|\Psi(0)\rangle = \sum_n c_n |E_n\rangle$$

$$|\Psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

$$\langle\Psi(t)|A|\Psi(t)\rangle = \sum_{n,n'} c_n c_{n'} e^{-i(E_{n'}-E_n)t/\hbar} A_{n,n'} \rightarrow Tr\left[\rho_T A\right]$$

$$\rho_T = \sum e^{-E_n/(k_B T)} |E_n\rangle\langle E_n|$$

Dissipation = absorption of classical waves by Black hole!



Policastro-Son-Starinets (2002):

Viscosity: absorption cross section of gravitons by black hole

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

= area of horizon (GR theorems)

Entropy density s: Bekenstein-Hawking
BH entropy = area of horizon

Universal viscosity-entropy ratio for CFT's with gravitational dual limited in large N by:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Quantum criticality and the dimension of viscosity ...

Viscosity: $\eta = f \tau_K$

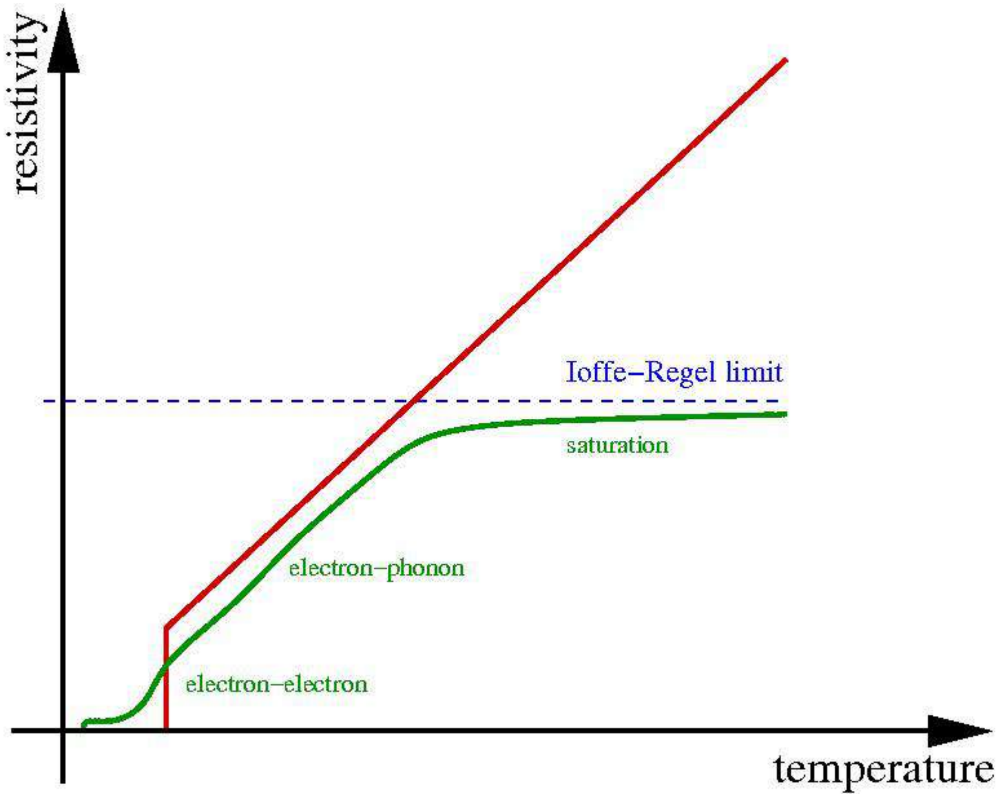
Free energy density QC system: $f = sT$

Planckian dissipation: $\tau_K = A \frac{\hbar}{k_B T}$

$$\frac{\eta}{s} = AT \frac{\hbar}{k_B T} = A \frac{\hbar}{k_B}$$

Large N SUSY Yang Mills: $A = \frac{1}{4\pi}$

The “Planckian” linear resitivity.



Strange metal transport: nearly momentum conserving.

van Heumen

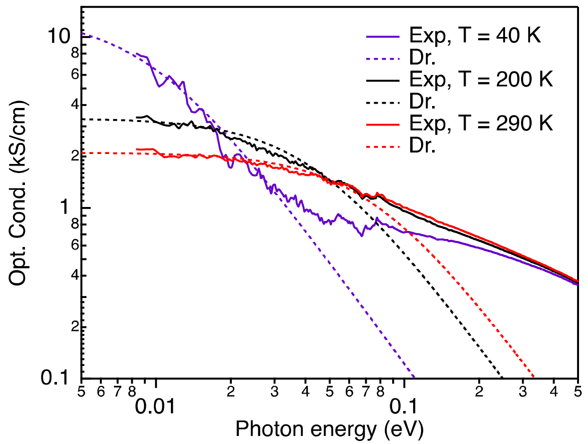
Optical conductivity: perfect Drude peak at low frequency followed by a gapped branch cut over the whole doping regime (single layer BISCO)!

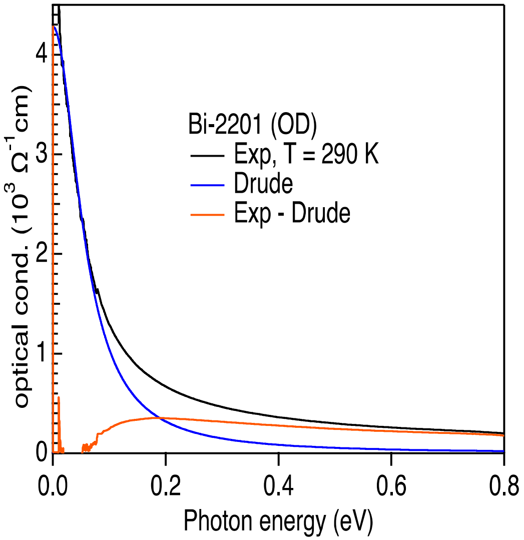
$$\sigma_1^D(\omega) = \frac{D_D \tau_L}{4\pi} \frac{1}{1 + (\omega\tau_L)^2}$$

$$\tau_L \simeq \frac{\hbar}{k_B T}$$

$$\sigma_1^{inc.}(\omega) = D_{inc} \mathrm{Im}(\sqrt{\Delta^2 - \omega^2} - i\Gamma)^\alpha$$

$$\alpha \simeq 2/3, \quad \Delta \simeq 80\mathrm{meV}$$





Solid state physics 101: Drude means "long lived total momentum"

Electrical current "***overlaps***" with momentum, conventionally: $\vec{J} = \frac{ne}{m}\vec{P}$

Nearly ***homogeneous*** space, momentum relaxation: $\frac{dP_{x,L}}{dt} + \frac{1}{\tau_L}P_{x,L} = eE_x$

$$\sigma_1(\omega) = \frac{\omega_p^2 \tau_L}{1 + (\omega\tau_L)^2} \qquad \omega_p = \sqrt{ne^2/m}$$

Cuprates (linear resistivity): $\tau_L \simeq \tau_\hbar = \frac{\hbar}{k_B T}$

"Planckian dissipation": the total *momentum relaxation rate* is at the "quantum limit of the production of heat"!

Holographic linear resistivity

(PRB 89, 2451161, 2014).

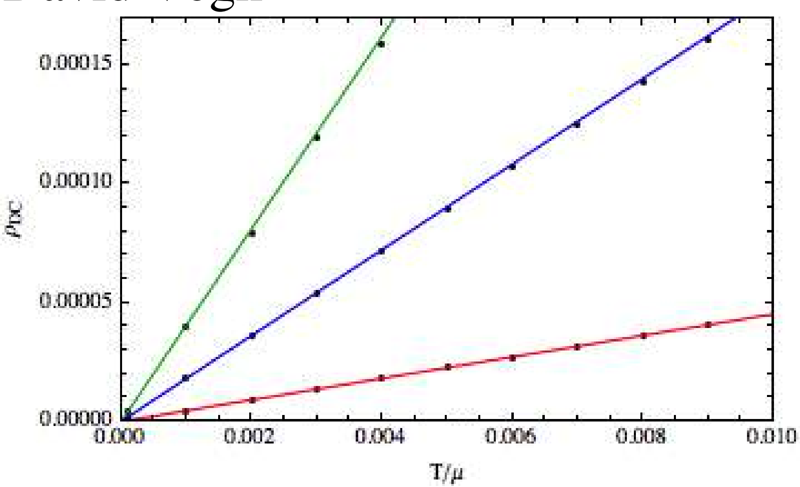
Richard Davison

Steve Gubser

David Vegh

"Conformal to AdS2" strange metal: Einstein-Maxwell-dilaton (consistent truncation), local quantum critical, marginal Fermi-liquid (3+1D), susceptible to holo. superconductivity, healthy thermodynamics: unique ground state, Sommerfeld thermal entropy.

Breaking of Galilean invariance (finite conductivities) due to quenched disorder: "massive gravity" = "Higgsing" space-like diffeomorphisms in the bulk !?



$$S=\frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\Big[\mathcal{R}-\frac{1}{4}e^{\phi}F_{\mu\nu}F^{\mu\nu}-\frac{3}{2}\partial_\mu\phi\partial^\mu\phi+\frac{6}{L^2}\cosh\phi-\frac{1}{2}m^2\left(\mathrm{Tr}\left(\mathcal{K}\right)^2-\mathrm{Tr}\left(\mathcal{K}^2\right)\right)\Big]$$

Explicit holographic construction explaining linear resistivity!

The secret of the linear resistivity (PRB 89, 2451161, 2014).

Davison

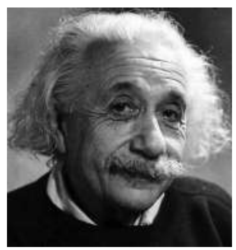
Hartnoll

Planckian dissipation = very rapid local equilibration: a hydrodynamical fluid is established before it realizes that momentum is non conserved due to the lattice potential (not true in Fermi-gas: Umklapp time is of order collision time).

Stokes

Resistivity in hydrodynamics

$$\rho(T) \propto \frac{1}{\tau_{rel}} = \frac{D}{l^2}$$



Einstein

Einstein relation:

$$D = \frac{\eta}{m_e n_e}$$

Sachdev

Son

Planckian viscosity

$$\eta = A \frac{\hbar}{k_B} s$$

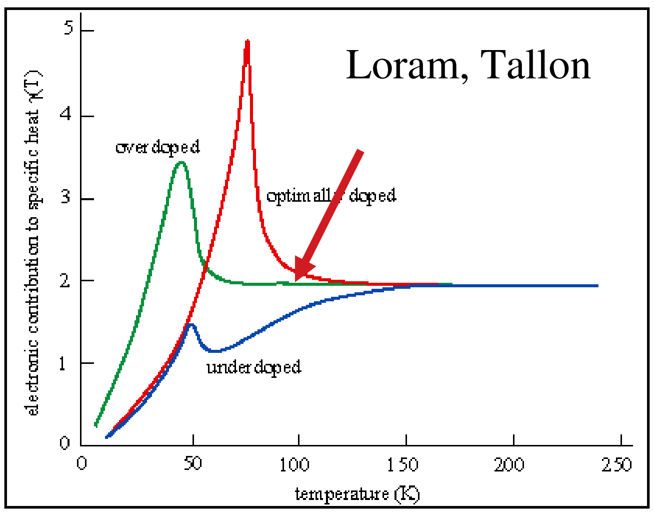
$$\rho(T) = \frac{1}{\omega_p^2 \tau_{rel}} = A \frac{\hbar}{\omega_p^2 l^2 m_e} \frac{S}{k_B}$$

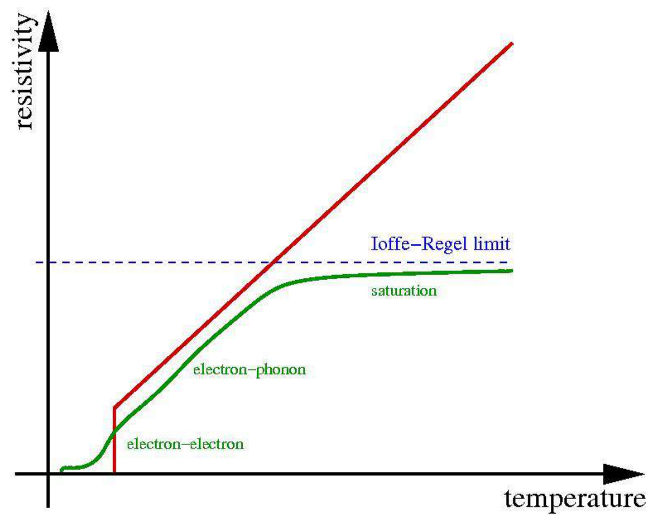
Entropy versus transport: optimal doping

Optimally doped

$$C = \gamma T \Rightarrow S = T/\mu$$

$$\rho \mu \frac{1}{\tau_{rel}} \mu S \mu T$$

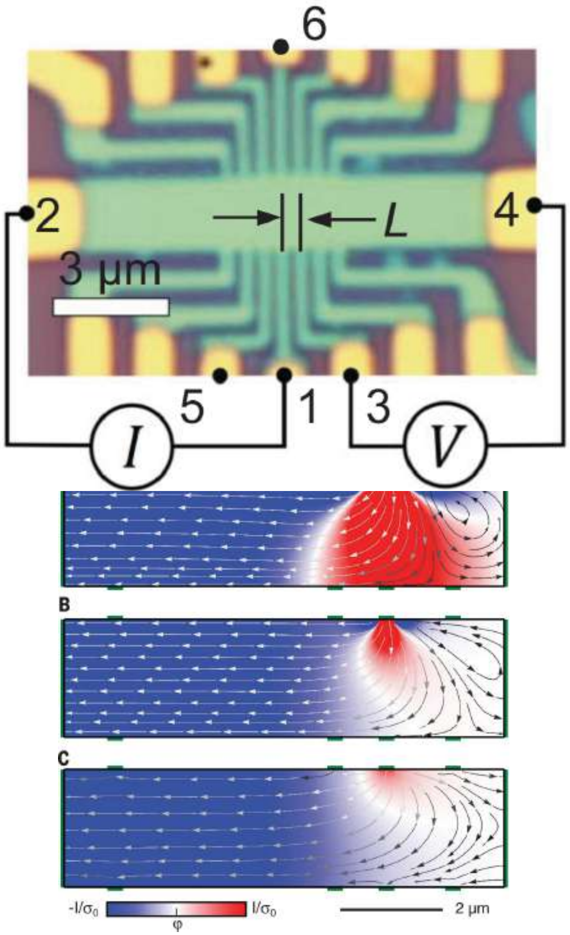




Plugging in numbers: "mean-free path" $l \approx 10^{-9} m$

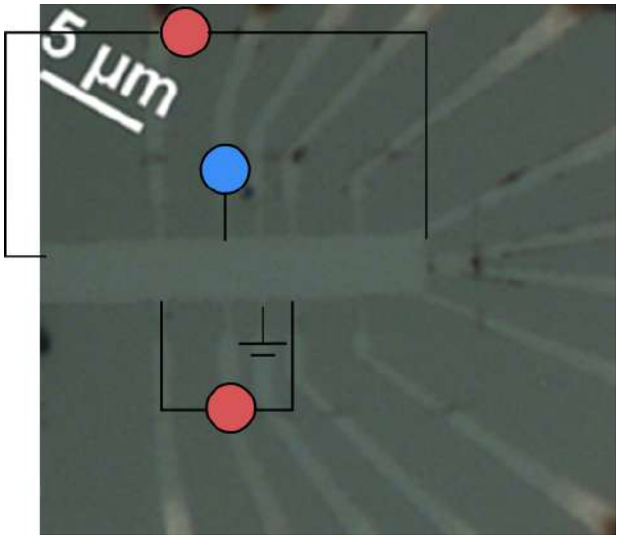
Nanoscale transport devices are required to look for hydro!

Geim



Graphene laminar whirls.

Cuprate quantum transport: fighting the chemistry!



Quantum turbulence: first principle vortex dissipation ..

Hua-Bi Zheng

Overview: lessons of holography.

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How it started …

analysis of TDH transcripts expressed in human fetal liver tissue showed complete skipping of exon 4 and either complete skipping or aberrant splicing of exon 6 (fig. S8). Given that exons 4 and 6 encode segments of the enzyme critical to its function and that truncation via the nonsense codon at amino acid 214 would also be predicted to yield an inactive variant, it appears that the human gene is incapable of producing an active TDH enzyme. Remarkably, all metazoans whose genomes have been sequenced to date, including chimpanzees, appear to contain an intact TDH gene (*14*). Unless humans evolved adaptive capabilities sufficient to overcome three mutational lesions, it would appear they are TDH deficient.

Human ES cells grow at a far slower rate than mouse ES cells, with a doubling time of 35 hours (*15*). Whether the slower growth rate of human ES cells reflects the absence of a functional TDH enzyme can perhaps be tested by introducing, into human ES cells, either a repaired human TDH gene or the intact TDH gene of a closely related mammal. That this strategy might work is supported by the expression in human cells of a functional form of the 2-amino-3-ketobutyrate-

String Theory, Quantum Phase Transitions, and the Emergent Fermi Liquid

Mihailo Čubrović, Jan Zaanen, Koenraad Schalm*

A central problem in quantum condensed matter physics is the critical theory governing the zero-temperature quantum phase transition between strongly renormalized Fermi liquids as found in heavy fermion intermetallics and possibly in high–critical temperature superconductors. We found that the mathematics of string theory is capable of describing such fermionic quantum critical states. Using the anti–de Sitter/conformal field theory correspondence to relate fermionic quantum critical fields to a gravitational problem, we computed the spectral functions of fermions in the field theory. By increasing the fermion density away from the relativistic quantum critical point, a state emerges with all the features of the Fermi liquid.

Quantum many-particle physics lacks a general mathematical theory to deal with fermions at finite density. This is known as the "fermion sign problem": There is no recourse to brute-force lattice models because the statistical path-integral methods that work for any bosonic quantum field theory do not work for finite-density Fermi systems.

Holographic fermions: large N UV sensitivity ….


Faulkner

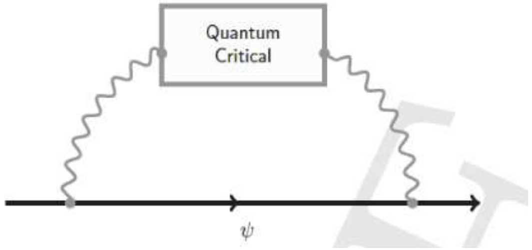

Polchinski

"Semi-holography" (arXiv:1001.5049):

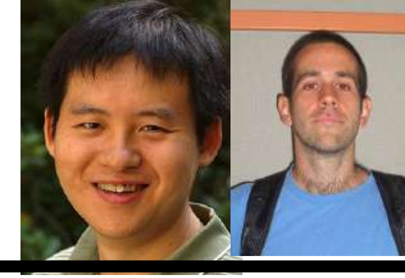
- Holographic fermions are 1/N suppressed, e.g. Dirac quantum mechanics in the bulk ("hbar^1").

- Strange metals: deconfining states large N YM theory. Upon lowering bulk mass "mesonic resonances" (confinement) develop = holographic quasiparticles.

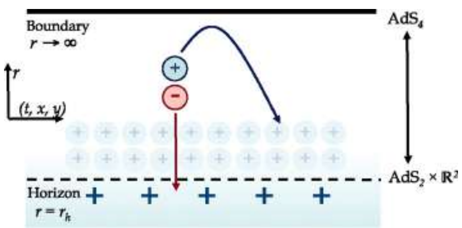
All vertex corrections are 1/N suppressed, quasiparticles are free fermions decaying by second order perturbation theory in "quantum critical heat bath", like claimed in e.g. marginal Fermi-liquids.



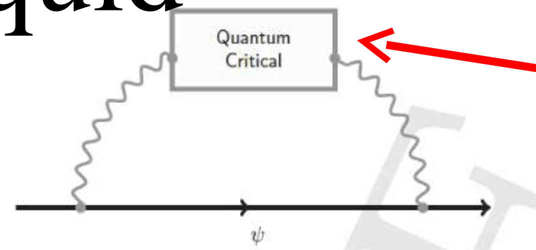
AdS/ARPES: the "RN" approaching the Fermi liquid



Liu McGreevy



Bulk: DW fermion gas and the horizon



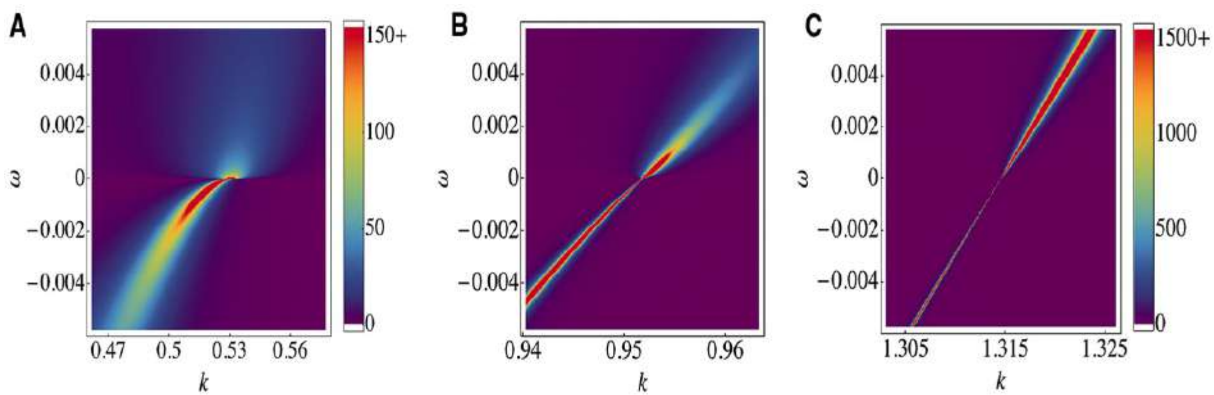
This is doing the collective (transport etc) work!

Boundary: would be fermions decaying in QC infrared

$$G_f(\omega, k) = \frac{1}{\omega - v_F(k - k_F) - \Sigma(k, \omega)}$$

$$\Sigma(k, \omega) \sim e^{i\phi_{k_F}} \omega^{2\nu_{k_F}}$$

$$2\nu_k \sim \sqrt{\frac{1}{\xi^2} + k^2}$$



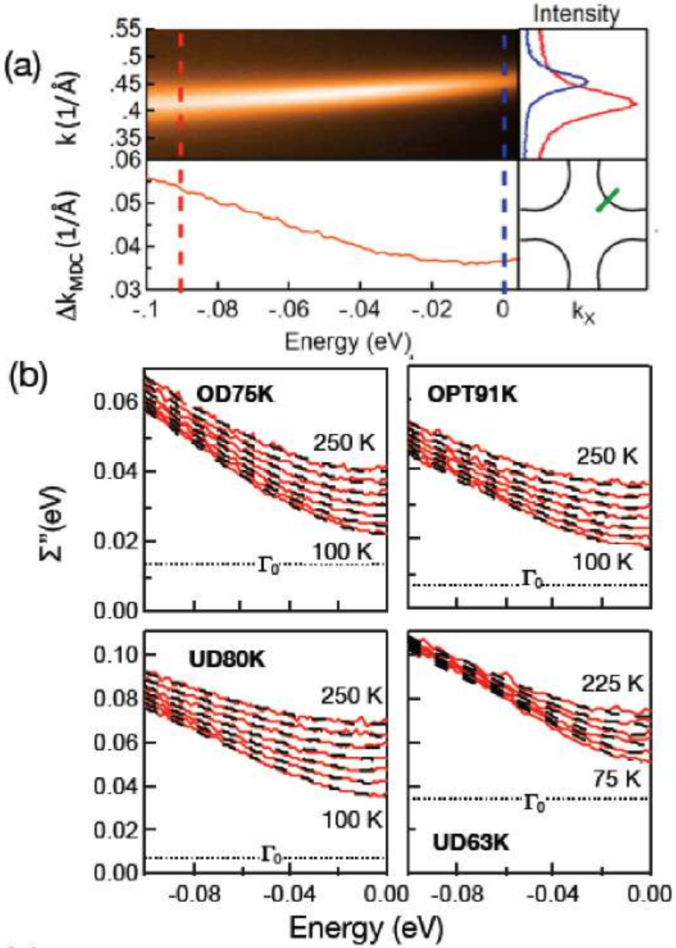
$2\nu_{k_F} < 1$ overdamped

$2\nu_{k_F} = 1$ Marginal FL

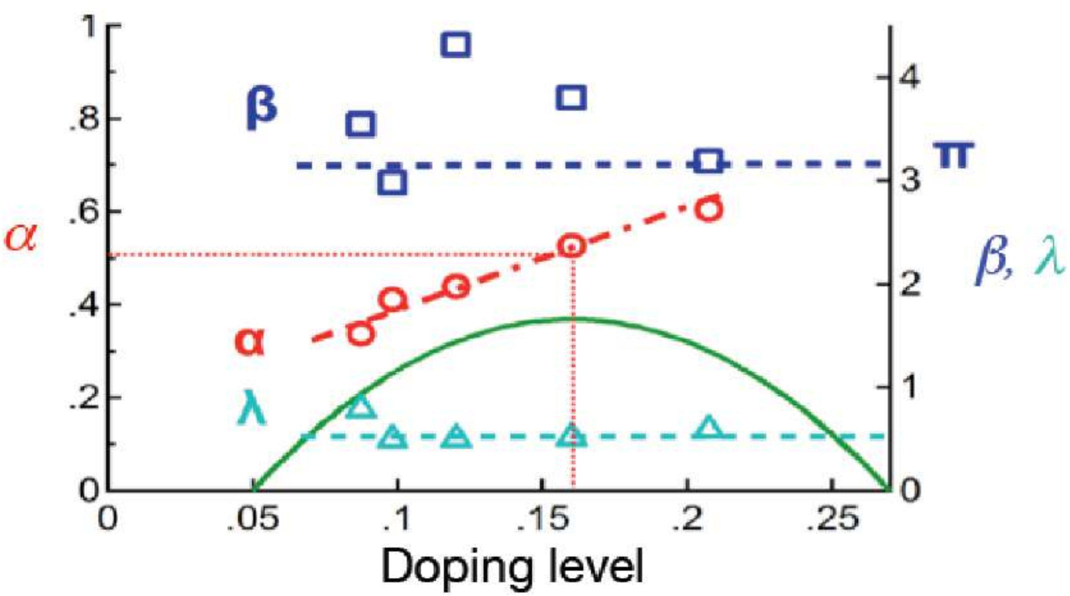
$2\nu_{k_F} > 1$ underdamped

Nodal fermion self-energies in the strange metal.

Dessau



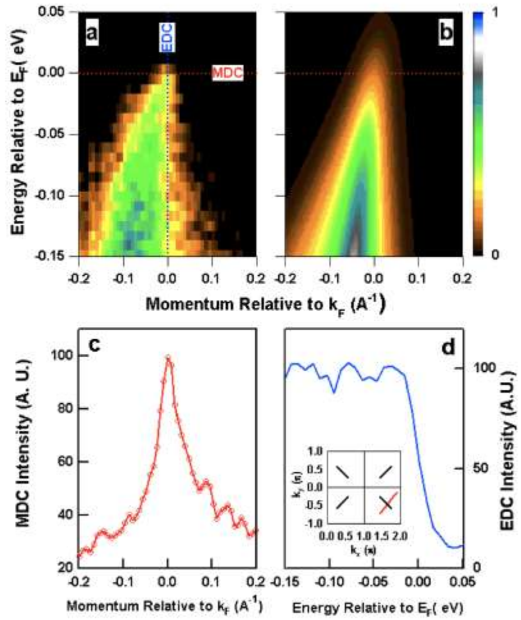
$$\Sigma''_{PLL}(\omega) = \Gamma_0 + \lambda \frac{\left[(\hbar\omega)^2 + (\beta k_B T)^2\right]^{\alpha}}{(\hbar\omega_N)^{2\alpha-1}}$$



arXiv:1509.01611

Cuprate ARPES: the “momentum-energy dichotomy”.

“Classic” anomaly: in underdoped cuprates in the antinodal (pseudogap) direction the MDC’s reveal sharp “marginal Fermi-liquid” Lorentzian peaks but the EDC’s are completely incoherent (e.g., Orgad et al., PRL 86, 4362, 2001).



This cannot be reconciled with a perturbative (self energy) fermion propagator!

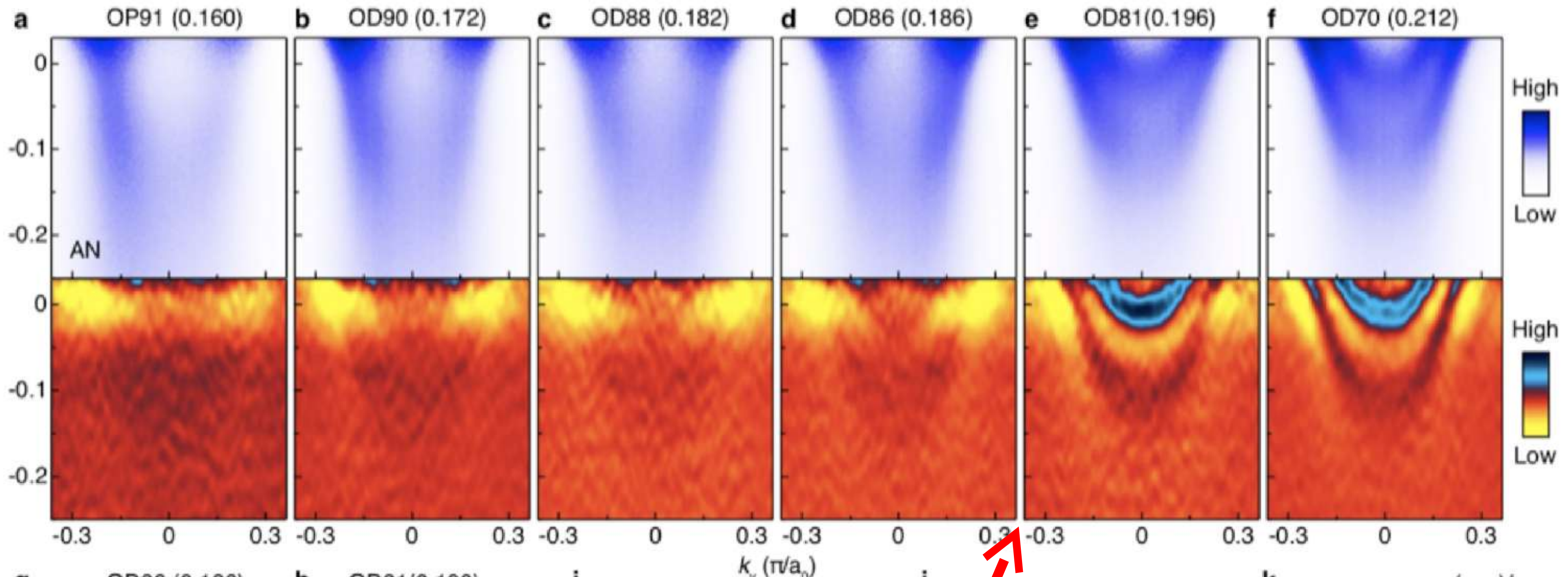
$$G_f(k,\omega) \neq \frac{1}{\omega - \varepsilon_k - \Sigma(k,\omega)}$$

The “first-order like” change at p_c.

Science 366, 1099 (2019)


Sudi Chen ZX Shen

Normal state 2212 antinodal ARPES as function of doping.



Strange metal: **incoherent**

Critical doping p_c =0.19

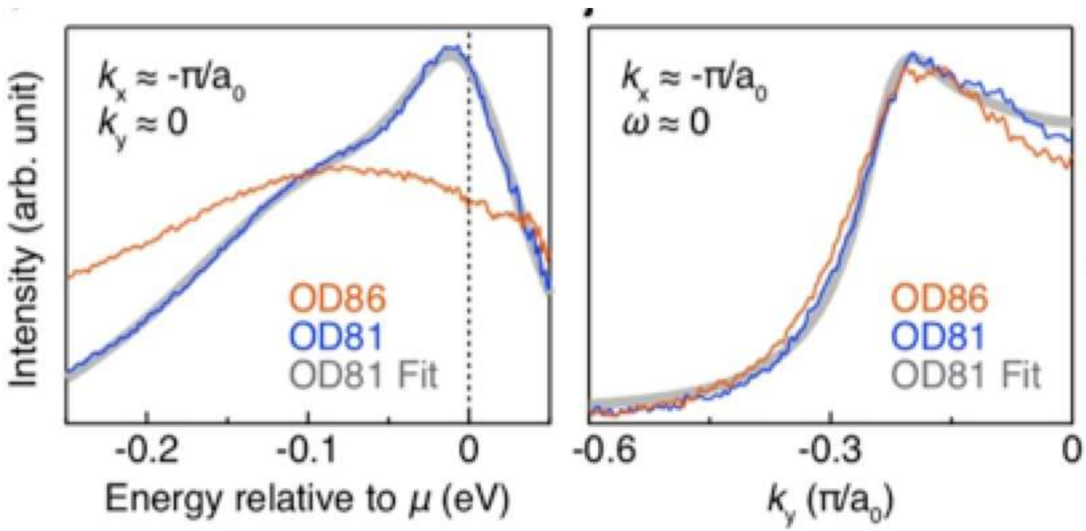
Reasonable quasiparticles !

The "first-order like" change at p_c.

Science 366, 1099 (2019)

Sudi Chen

ZX Shen



"The UV first order transition failing in the IR"

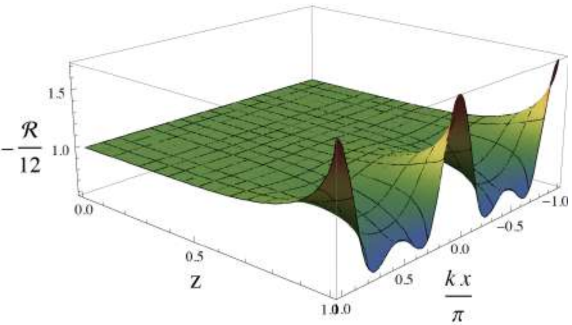
EDC's: perfect fit obtained using the industry standard "nodal" self energy for overdoped (Tc = 81 K) metal while the Tc= 86 K metal is completely incoherent.

MDC's: industry standard self-energies fit well on both sides of p_c.

Mark's talk: same kind of "space versus time" weirdness is at work at the nodes!

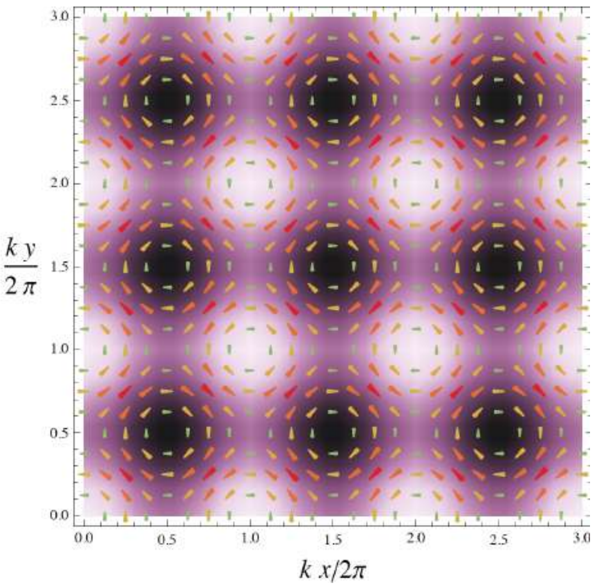
Complex horizons: intertwined charge-current-parity-SC order.

Rong-Gen Cai, Li Li, Yong-Qiang Wang, JZ, PRL 119, 181601 (2017), see also Andrade et al., Nature Physics 14, 1049 (2018).

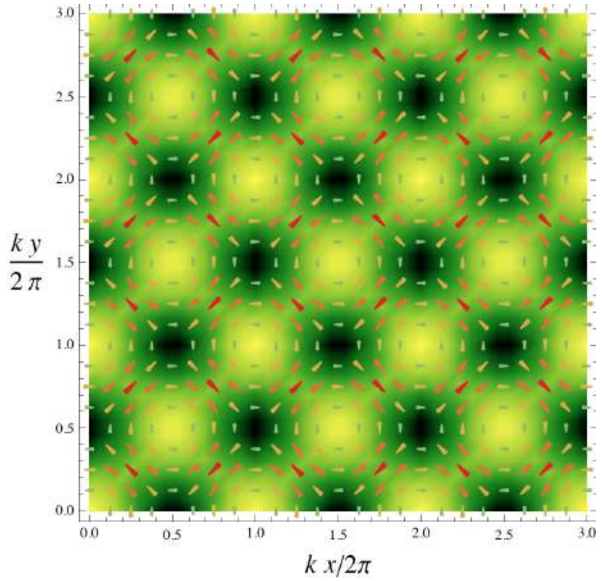


Black-hole hair, rasta style …

Charge and currents



Superconductivity



Charge order intertwined with parity breaking, spontaneous currents and pair density wave order.

Questions to experiment.

1. Are the cuprate strange metals "generalized Fermi-liquids", metallic phases with "anomalous covariant scaling dimensions?

Barrage of recent results suggesting the existence of an "underdoped" and "overdoped" strange metal phase.

Shen group (ARPES, also Tallon spec. heat): Science 366, 1099 (2019); Hussey group (magnetotransport): Nature 595, 661 (2021) (& to be published, also Sebastian et al., to be published)

2. Evidence for "covariant scaling with anomalous dimensions"?

Thermodynamics, Abbamonte's EELS, "conformal tail" in opt. conductivity (van der Marel 2003), Dessau's "nodal liquid" (ARPES): more dynamical linear response info in high demand!

3. Finite temperature transport and the Planckian dissipation.

Conclusions: lessons of holography.

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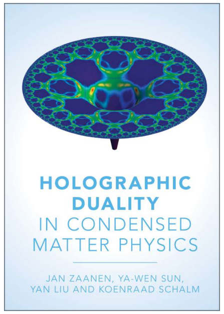
2. Finite temperature: bulk black holes encoding for the "Planckian" quantum thermalization.

3. Don't trust the holographic oracle! Fermions and the large N UV sensitivity …

Further reading

1. The mysteries of high Tc superconductivity: quantum supreme matter? Keimer et al., Nature 518, 179 (2015).

2. AdS/CFT, black holes computing the physics of dense entanglement: "Holographic duality in condensed matter physics", JZ, Liu, Sun and Schalm, CUP 2015.



3. Holography and Planckian dissipation: JZ, arXiv:1807.19951

4. Lecture notes "quantum supreme matter": JZ, arXiv:xxxx.xxxxx

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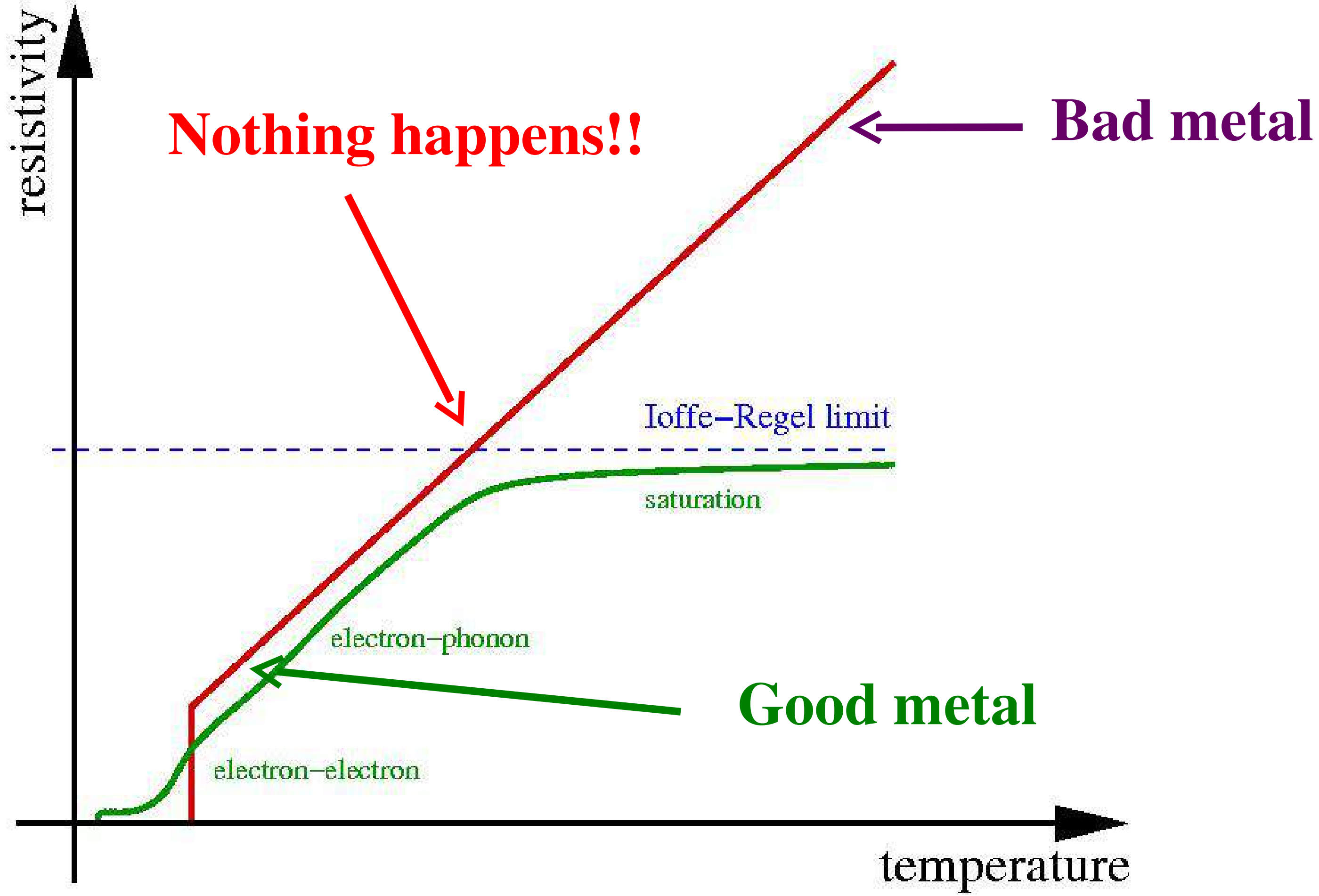
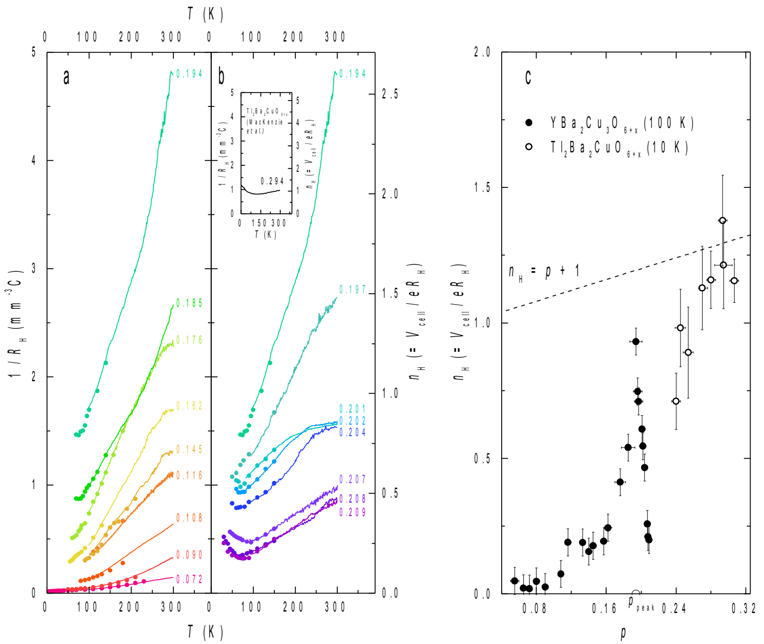
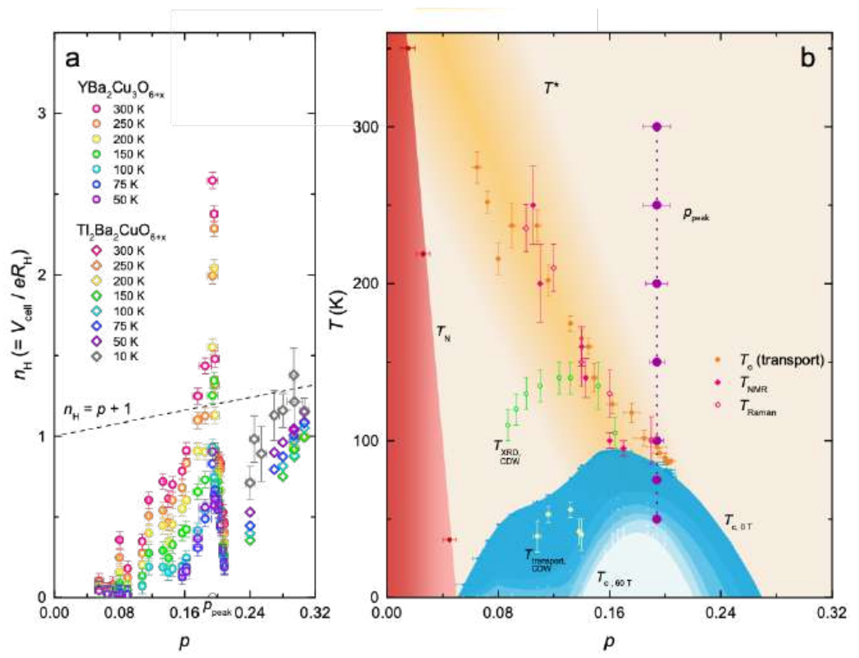
Divine resistivity



The news: doping dependence of Hall effect in $YBa_2Cu_3O_{7-y}$.

Sebastian





The news: the Hall relaxation time jumps at p_c!

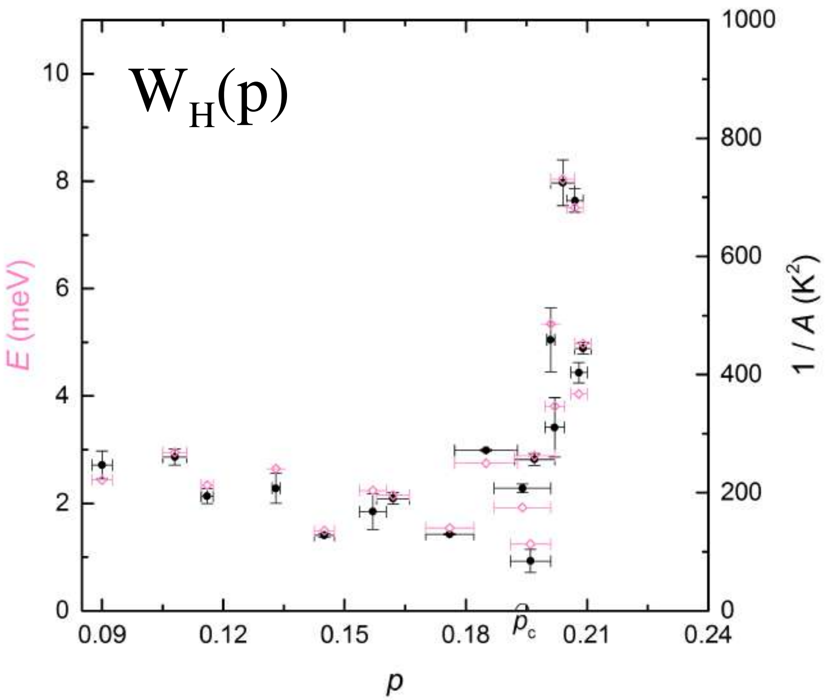
Measured Hall angle:

$$\Theta_H = \omega_c \tau_T \qquad \omega_c = \frac{eB}{m}$$

The quadratic temperature dependence implies an energy scale, e.g. Fermi liquid:

$$\frac{1}{\tau} \simeq \frac{k_B T^2}{E_F \hbar} = \frac{k_B T}{E_F} \frac{1}{\tau_\hbar}, \quad \tau_\hbar = \frac{\hbar}{k_B T}$$

Hall relaxation rate: $$\frac{1}{\tau_T} = \frac{k_B T}{W_H(p)} \frac{1}{\tau_\hbar}$$



Up to the critical doping W_H is doping independent, to shoot up in the overdoped regime: a DC quantity picks up the ARPES "first order" T=0 transition!

Eigenstate Thermalization hypothesis (Srednicki, Deutscher, 90's)

"Any local observer gets overwhelmed by the enormous amount of quantum info in the physical world to such an extent that he/she does not know better than that everything becomes a thermal state at long times."

Precise formulation:

$$|\Psi(0)\rangle = \sum_n c_n |E_n\rangle$$

$$|\Psi(t)\rangle = \sum_n c_n e^{-iE_n t/\hbar} |E_n\rangle$$

$$\langle\Psi(t)|A|\Psi(t)\rangle = \sum_{n,n'} c_n c_{n'} e^{-i(E_{n'}-E_n)t/\hbar} A_{n,n'} \to Tr\left[\rho_T A\right]$$

$$\rho_T = \sum e^{-E_n/(k_B T)} |E_n\rangle\langle E_n|$$