

# Charge transport in gapless, pinned charge density waves

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Strange metals: from the Hubbard model to AdS/CFT  
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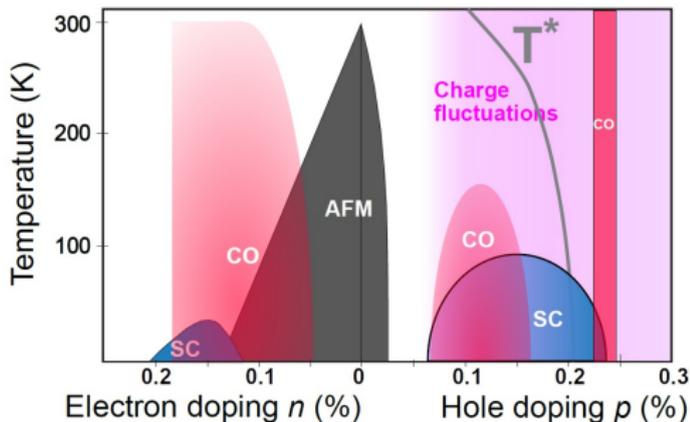


## References and acknowledgments:

- *'Effective and holographic theories of strongly-correlated phases of matter with broken translations'* [[ARXIV: 2203.03298](#)] with M. Baggioli.
- *'Damping of pseudo-Goldstone fields'* [[PHYS. REV. LETT. 128, 141601 \(2022\)](#)], with L. Delacrétaz and V. Ziogas.
- *'Universal relaxation in a holographic metallic density wave phase'* [[PHYS. REV. LETT. 123, 211602 \(2019\)](#)], with A. Amoretti, D. Areán, D. Musso.
- *'Bad Metals from Density Waves'* [[SciPost Phys. 3, 025 \(2017\)](#)] and *'Theory of hydrodynamic transport in fluctuating electronic charge density wave states'* [[PHYS. REV. B 96, 195128 \(2017\)](#)] with L. Delacrétaz, S. Hartnoll and A. Karlsson.
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- All references in [\[MAGENTA\]](#) have hyperlinks.

**Spontaneous breaking of translations** across the phase diagram of cuprates and other strange metals: various shades of incommensurate charge density waves.

- Expected on theoretical grounds since early days [Zaanen & Gunnarson, PRB'89], [Machida, Phys. C: Superconductivity'89], arguments for electronic liquid crystal phases in doped Mott insulators [Kivelson et al, Nature'98]. Doped holographic Mott insulators [Andrade et al, Nat. Phys.'18].



CREDIT: [FRANO ET AL, ARXIV: 2102.09525]

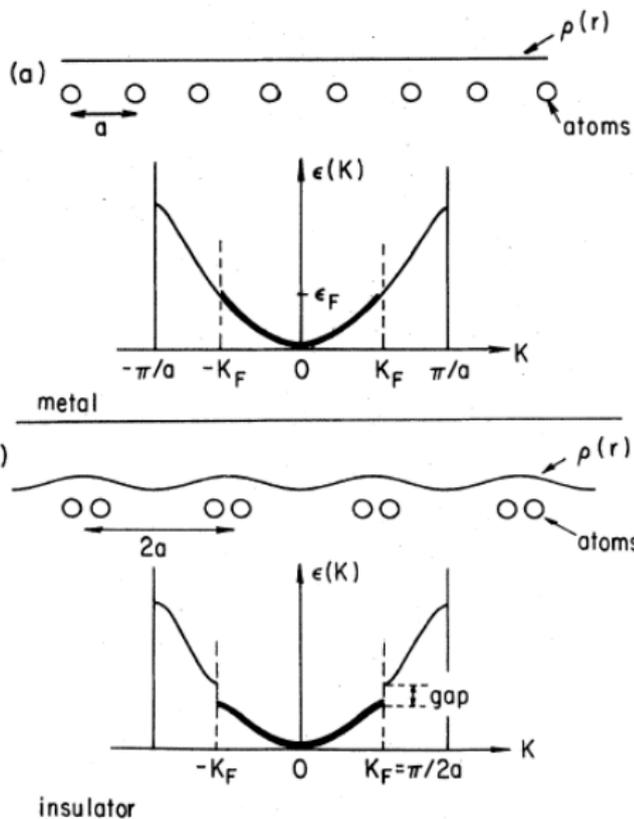
- Well-established in underdoped cuprates [Tranquada et al, Nature'95].
- More recent discovery on the **overdoped** side [Arpaia et al, Science'19], see [Arpaia and Ghiringhelli, J. Phys. Soc. Jpn.'21] for a review.
- Magnetism all the way to the pseudogap critical point, [Frachet et al., Nat. Phys. '20]

Weakly-coupled, quasi-particle based mechanism in quasi one-dimensional materials: **Peierls instability**, [GRÜNER, RMP'88].

Gap opens, modulated density of states energetically favored

$$\rho(x) = \rho_0 + \delta\rho \cos(k_{cdw}x + \varphi^x)$$

$\varphi^x$ : Goldstone mode ('phason') of **spontaneously broken translations**.



CREDIT [GRÜNER, RMP'88]

Low frequencies, weak disorder: pseudo-Goldstone mode

$$f = \dots + \frac{\kappa}{2} (\partial_x \varphi^x)^2 + \frac{\kappa}{2} m_\varphi^2 (\varphi^x)^2$$

- Relaxed dynamics for  $\varphi^x$ :

$$\partial_t^2 \varphi^x + \Gamma \partial_t \varphi^x + \omega_o^2 \varphi^x = 0$$

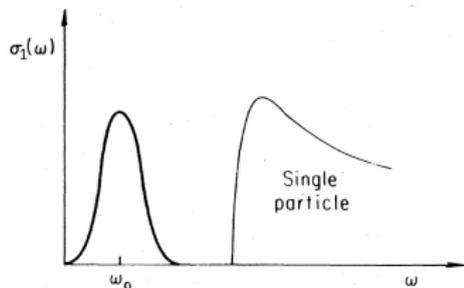
Weak disorder:  $\Gamma, \omega_o \ll \Delta$  the single particle gap  $\Rightarrow$  **pseudo-Goldstone remains light.**

- CDW is **pinned** [GRÜNER, RMP'88]

$$\sigma(\omega) = \left( \frac{ne^2}{m^*} \right) \frac{-i\omega}{-i\omega(\Gamma - i\omega) + \omega_o^2}$$

$\Gamma$ : momentum relaxation rate.

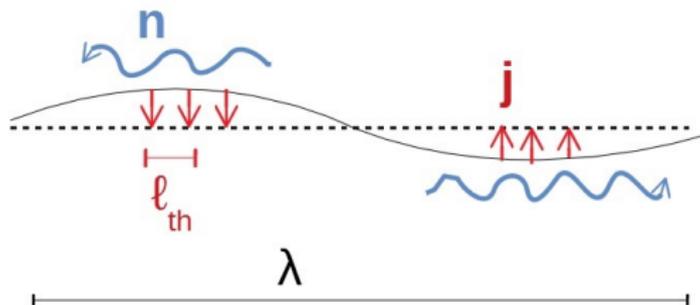
$\omega_o^2 \equiv \kappa m_\varphi^2 / (m^* n)$ : pinning frequency.



CREDIT: ADAPTED FROM [GRÜNER, RMP'88]

Transfer of spectral weight. Pinning short-circuits the DC conductivity: **insulator**. Gap = no available relaxational channel for the Goldstone.

- This mechanism requires (quasi) one-dimensional Fermi surfaces with weakly-coupled quasiparticles, and typically does apply in many strongly-correlated materials.
- Instead, Mott physics, anti-ferromagnetic fluctuations, etc. No hard gap: **gapless low-energy excitations** on top of the Goldstone mode.
- Rather than focusing on a specific material, I want to investigate on general grounds **charge transport in pinned, gapless, strongly-correlated charge density wave states** (see [S. KRIVUN'S TALK] for thermoelectric effects).



- Strong correlations imply **short equilibration scales**  $\tau_{eq} \sim 1/T$  ('Planckian'? See [R. DAVISON'S TALK] for a holographic illustration of this), which justify the use of effective field theory methods for the low-energy dynamics.

- EFTs rely on the **symmetries** of the system  $\Rightarrow$  **conservation equations**

$$\partial_t n + \partial_i j^i = 0$$

- and on an **expansion in gradients**  $\tau_{eq} \partial_t \ll 1$ ,  $l_{th} \partial_x \ll 1 \Rightarrow$  **constitutive relations** for vevs of currents in the thermal equilibrium state.

- The main result is that compared to [GRÜNER, RMP'88] an **extra transport coefficient** is needed, which governs the **(inverse) lifetime of the pseudo-Goldstone**.
- It is fixed by its mass and a diffusivity that characterizes sound attenuation in the clean system (no disorder)

$$\Omega = m_\varphi^2 D_\varphi$$

- It is a direct consequence of the existence of a bath of thermal excitations, into which the Goldstone can relax.

- EFTs are built starting from **symmetries**: tricky to write them when symmetries are **approximate**. In fact we missed this coefficient when we wrote an EFT for pinned CDWs, [DELACRÉTAZ ET AL, PRB'17].
- The need for this relaxed transport coefficient  $\Omega$  was made obvious when we tried to check the EFT using **holographic methods** [AMORETTI ET AL, PRL'19] (see also [DONOS ET AL, JHEP'19], [DONOS ET AL, CLASS.QUANT.GRAV.'20], [ANDRADE ET AL, JHEP'21]).
- We then went back to the EFT and showed it follows from **consistency of coupling the static partition function to external sources**, [DELACRÉTAZ ET AL, PRL'22] (also shown to follow from **positivity of entropy production**, [ARMAS ET AL, ARXIV: 2112.14373]).<sup>†</sup> Not an artifact of large  $N$  or of specific holographic setups!

<sup>†</sup>In [ARMAS ET AL, ARXIV: 2112.14373], other transport coefficients are also derived, but appear to renormalize static susceptibilities and so play a less dominant role for the purposes of this talk.

- The AC conductivity has a more complicated  $\omega$  dependence:

$$\sigma(\omega) = \left( \frac{ne^2}{m^*} \right) \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

**Drude peak** if  $\omega_o$  sufficiently small compared to  $\Omega$ .

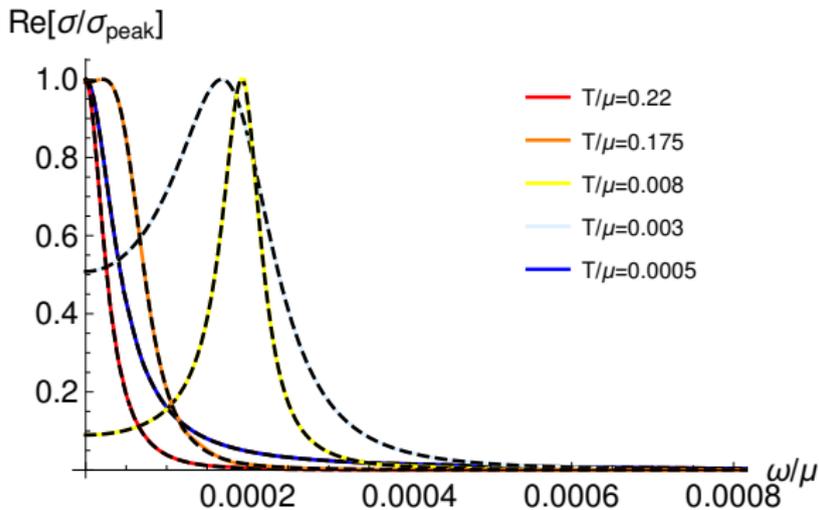
- Nonzero dc resistivity:**

$$\rho_{dc} = \frac{m^*}{ne^2} \left( \Gamma + \frac{\omega_o^2}{\Omega} \right) = \frac{m^*}{ne^2} \left( \Gamma + \frac{v^2}{D_\varphi} \right), \quad v^2 = \frac{\kappa}{m^*n}$$

The second term is **independent on the strength of disorder/explicit translation symmetry breaking** to leading order.

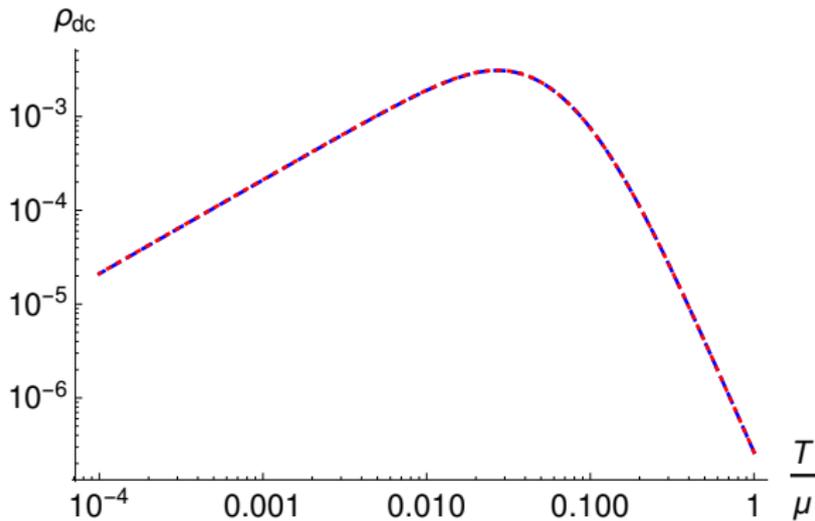
- Reminiscent of an **Einstein relation**, as here the thermal diffusivity:

$$D_T \sim D_\varphi$$



The holographic result for the AC conductivity in a phase that breaks translations pseudo-spontaneously matches the EFT prediction extremely well.<sup>†</sup>

<sup>†</sup>Accounting for the underlying Lorentz invariance of the holographic system, etc.



The resistivity dominated by the pseudo-Goldstone contribution<sup>†</sup>

$$\rho_{dc} \simeq \frac{m^* \omega_o^2}{ne^2 \Omega}$$

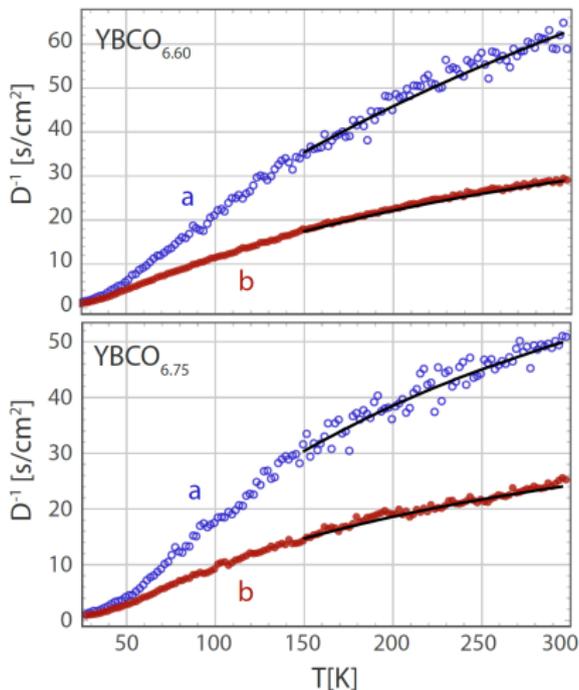
$D_\varphi$  controlled by horizon quantities, [AMORETTI ET AL, JHEP'19]: **the Goldstone couples to the black hole horizon**, which provides the bath of thermal excitations into which it relaxes. 'holographic black hole membrane paradigm' [IQBAL & LIU, PHYS.REV.D'09], [DONOS & GAUNTLETT, JHEP'14].

<sup>†</sup> Accounting for the underlying Lorentz invariance of the holographic system, etc.

- In strongly-correlated materials, generally expect **diffusivities to saturate a lower bound** [KOVTON, SON & STARINETS, PRL'05], [HARTNOLL, NAT. PHYS.'14]

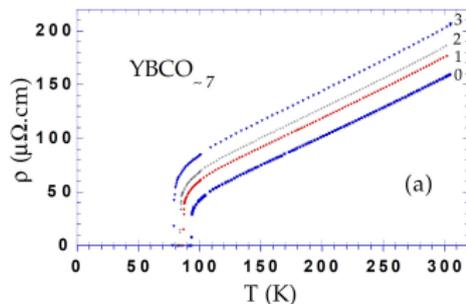
$$D \gtrsim \frac{\hbar v^2}{k_B T}$$

Eg thermal diffusivity in the strange metal regime [ZHANG ET AL, PNAS'17].

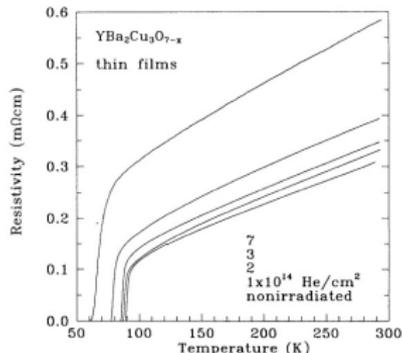


- Yields a  $T$ -linear resistivity, **slope independent on the strength of disorder/explicit translation symmetry breaking** to leading order

$$D_\varphi \simeq \frac{\hbar v^2}{k_B T}, \quad \rho_{dc} \simeq \frac{m^*}{ne^2} \frac{v^2}{D_\varphi} + O(\Gamma) \sim T$$



CREDIT: [RULLIER-ALBENQUE ET AL, EUR.PHYS.LETT'00]



CREDIT: [WALKER ET AL, PHYS REV B'94]

Emphasis on the independence of the slope on disorder: same slope for across different overdoped cuprates, in spite of varying degree of disorder

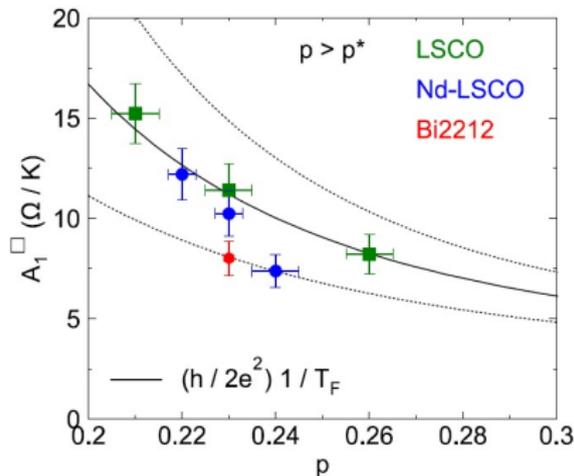
- Extract the  $T$ -linear component of the resistivity

$$\rho \simeq \rho_0 + A_1 T + \dots, \quad A_1^\square = A_1/d$$

$$\rho \simeq \frac{m^*}{ne^2\tau}, \quad \tau = \frac{\hbar}{\alpha k_B T}$$

$$A_1^\square = \alpha \frac{h}{2e^2} \frac{1}{T_F}, \quad T_f = \frac{\pi \hbar^2}{k_B} \frac{nd}{m^*}$$

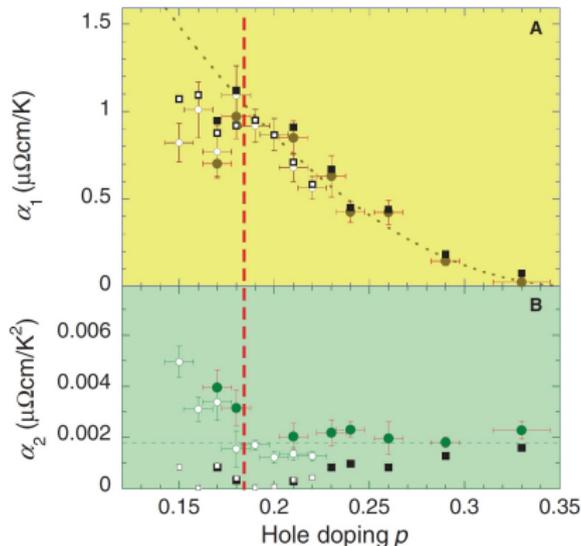
- If we had a simple Drude model, expect that  $1/\tau \sim g^2$ , highly dependent on the strength of disorder.



CREDIT: [LEGROS ET AL, NAT. PHYS.'19]

- Two distinct temperature dependencies in transport [COOPER ET AL SCIENCE'09], [PUTZKE ET AL NATURE PHYSICS'21], [AYRES ET AL ARXIV: 2012.01208]

$$D_\varphi \sim \frac{v^2 \hbar}{\alpha k_B T}, \Gamma \sim \gamma_0 + \gamma_2 T^2 \quad \Rightarrow \quad \rho_{dc} \sim \frac{m^*}{ne^2} \left( \gamma_0 + \frac{k_B \alpha}{\hbar} T + \gamma_2 T^2 \right)$$



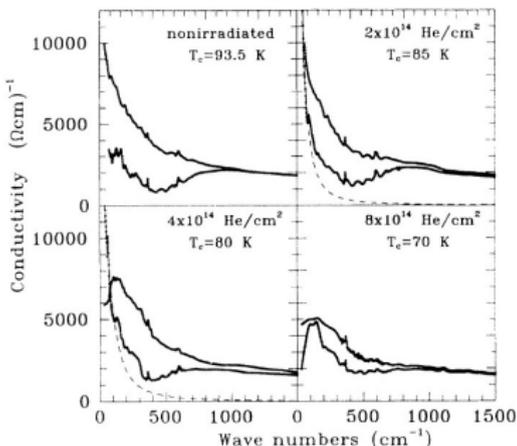
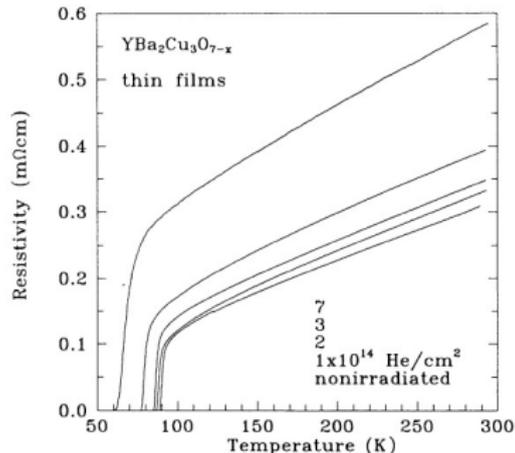
CREDIT: [COOPER ET AL SCIENCE'09]

- Upon increasing disorder, the **Drude peak** in the strange metal regime is **transferred to nonzero frequencies** in He-irradiated  $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ .

- Reproduced by the EFT prediction for the ac conductivity when **pinning  $\omega_o$  is stronger than damping  $\Omega$**

$$\sigma(\omega) = \left( \frac{ne^2}{m^*} \right) \frac{\Omega - i\omega}{(\Omega - i\omega)(\Gamma - i\omega) + \omega_o^2}$$

- Same transfer of spectral weight observed in the strange metal regime as  $T$  increases [HUSSEY ET AL, PHILOS. MAG.'04], [DELACRÉTAZ ET AL, SCIPOST PHYS.'17]: consistent with  $\Omega \sim \omega_o^2 D_\varphi \sim \omega_o^2 / T$ .



[BASOV ET AL, PHYS REV B'94]

- EFTs and holographic methods used in conjunction to arrive at general statements on transport in strongly-correlated phases of quantum matter.
- Example from charge transport in pinned, gapless charge density wave phases: nonzero resistivity from relaxation of pseudo-Goldstone into bath of thermal excitation

$$\Omega = m_\varphi^2 D_\varphi$$

- In holography,  $D_\varphi$  is controlled by the black hole horizon.
- Appealing features for charge transport in cuprate strange metals.

THANKS!