The Higgs/Amplitude Mode in Holography

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Based on 2205.06294 with C. Pantelidou

Outline

- Introduction
- Phase Transitions in Holography
- The Higgs/Amplitude Mode
- Conclusions

Motivation

- At strong coupling often no quasiparticles
- Conserved charges and light Goldstone modes can dominate at long wavelengths
- Correlation length diverges during a transition
- Use holography to carry out microscopic computations

(Conformal) Field Theory Setup

- Relativistic field theory (with global $U(1)$) at finite T and zero charge
- Charged operator $\mathcal{O}_\psi$ transforms as $\mathcal{O}_\psi \to e^{-iq\alpha}\,\mathcal{O}_\psi$
- Phase transition with $\langle \mathcal{O}_\psi \rangle \neq 0$ at $T < T_c$
- Couple to perturbative external gauge field A_μ and scalar source λ_ψ

$$\delta S = \int d^n x \left(J^\mu \, \delta A_\mu + \mathcal{O}^*_\psi \, \delta\lambda + \mathcal{O}_\psi \, \delta\lambda^* \right)$$

Finite Temperature

- Generating function $W[A_\mu, \lambda, \lambda^*]$ depends on external gauge field A_μ and complex source λ
- Functional differentiation gives the VEVs

$$\langle J^\mu \rangle = i\frac{\delta W}{\delta A_\mu}, \qquad \langle \mathcal{O}_\psi \rangle = i\frac{\delta W}{\delta \lambda^*}$$

- Invariance under gauge transformations $\delta A_\mu \to \partial_\mu \delta\Lambda, \quad \delta\lambda = -\, i\, q\, \lambda\, \delta\Lambda$ yields the current (non)-conservation Ward identity

$$\nabla_\alpha \langle J^\alpha \rangle = iq\left(\langle \mathcal{O}_\psi \rangle \lambda^* - \langle \mathcal{O}_\psi^* \rangle \lambda\right)$$

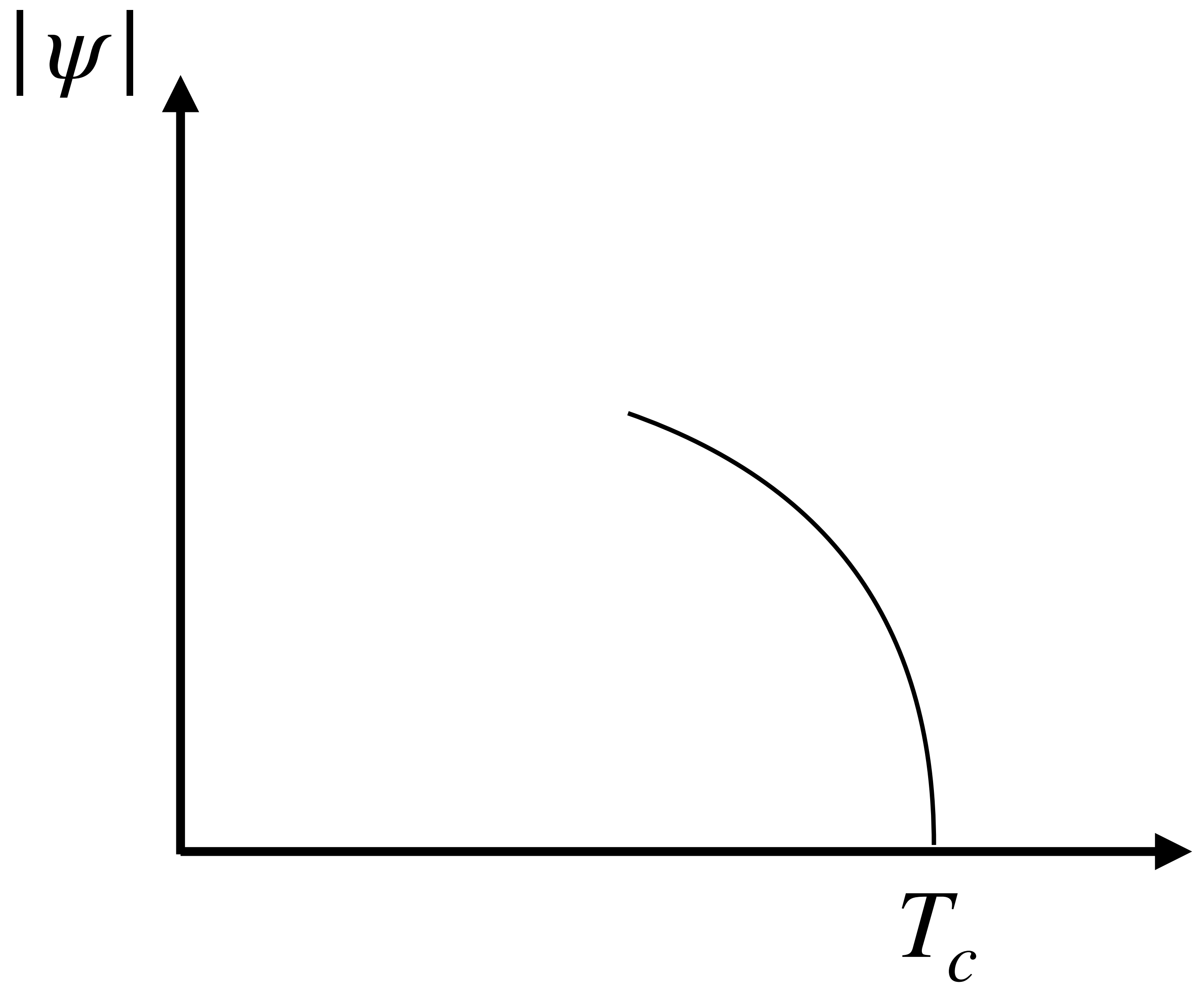
- Invariance under coordinates transformations yields the stress tensor Ward identity

$$\nabla_\mu \langle T^{\mu\nu} \rangle = F^{\nu\mu}\, \langle J_\mu \rangle + \nabla^\nu \lambda\, \langle \mathcal{O}_\psi^* \rangle + \nabla^\nu \lambda^*\, \langle \mathcal{O}_\psi \rangle$$

Ginzburg - Landau

$$F[\psi] = \int d^3x \left(\frac{\hbar^2}{2m(T)} |\vec{\nabla}\psi|^2 + a'(T)|\psi|^2 + \frac{b'(T)}{2}|\psi|^4 \right) + F_n(T, \ldots)$$

$$m(T) = m + \cdots \qquad b'(T) = b' + \cdots \qquad a'(T) = T_c\,\alpha\,(1 - T/T_c) + \cdots$$



- Write leading order terms for free energy $F[\psi]$ in terms of the order parameter ψ
- Minimise free energy at equilibrium
  - $\psi = 0$ when $T > T_c$
  - $|\psi|^2 = -\frac{T_c\alpha}{b'}(1 - T/T_c)$ when $T < T_c$
  - Second order transition $\Delta F/\mathrm{vol} = -\frac{\alpha^2}{2b'}(T - T_c)^2$

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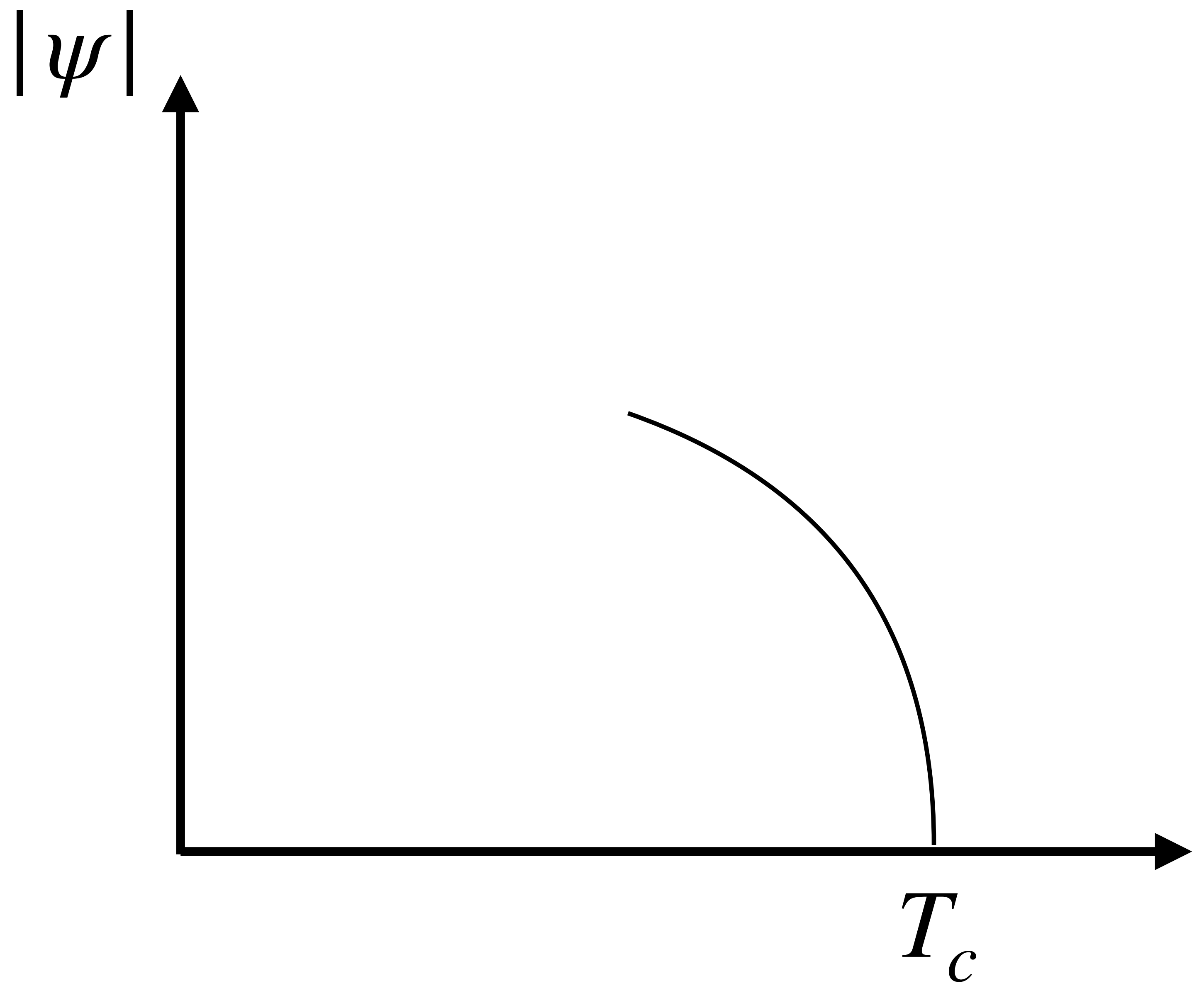


- Extremisation does not fix the phase ϑ of ψ
  - Goldstone mode due to spontaneous $U(1)$ symmetry breaking
- When slightly off equilibrium order parameter governed by

$$\partial_t \psi = -\Gamma \frac{\delta F}{\delta \psi^*} + \text{thermal noise} + \text{sources}$$

Halperin, Hohenberg

  - Simple gapped pole related to amplitude of order parameter with $\omega_{rel} \propto T_c - T$

Hydrodynamics at $T \ll T_c$

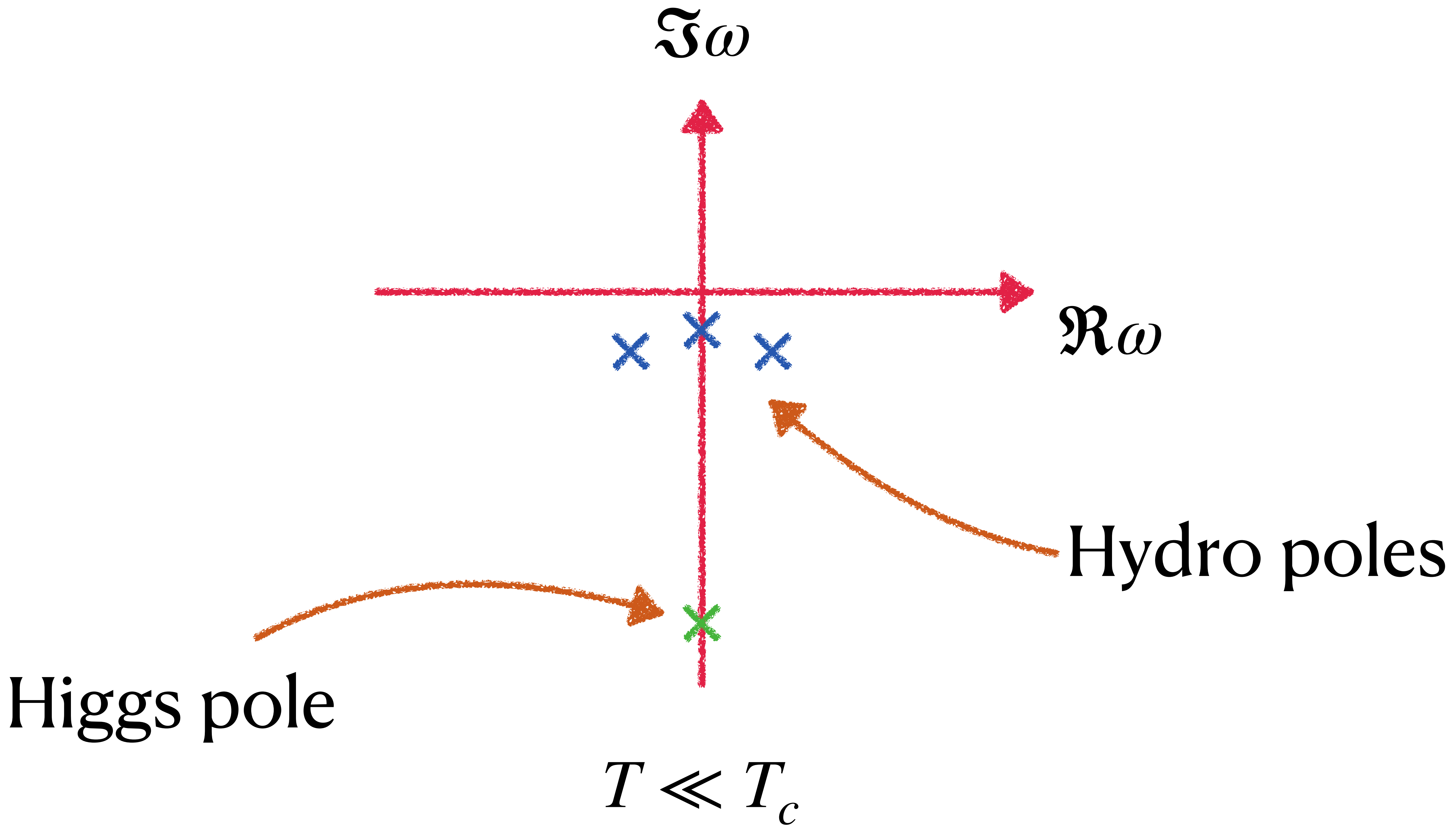
Parametrise massless collective dof:

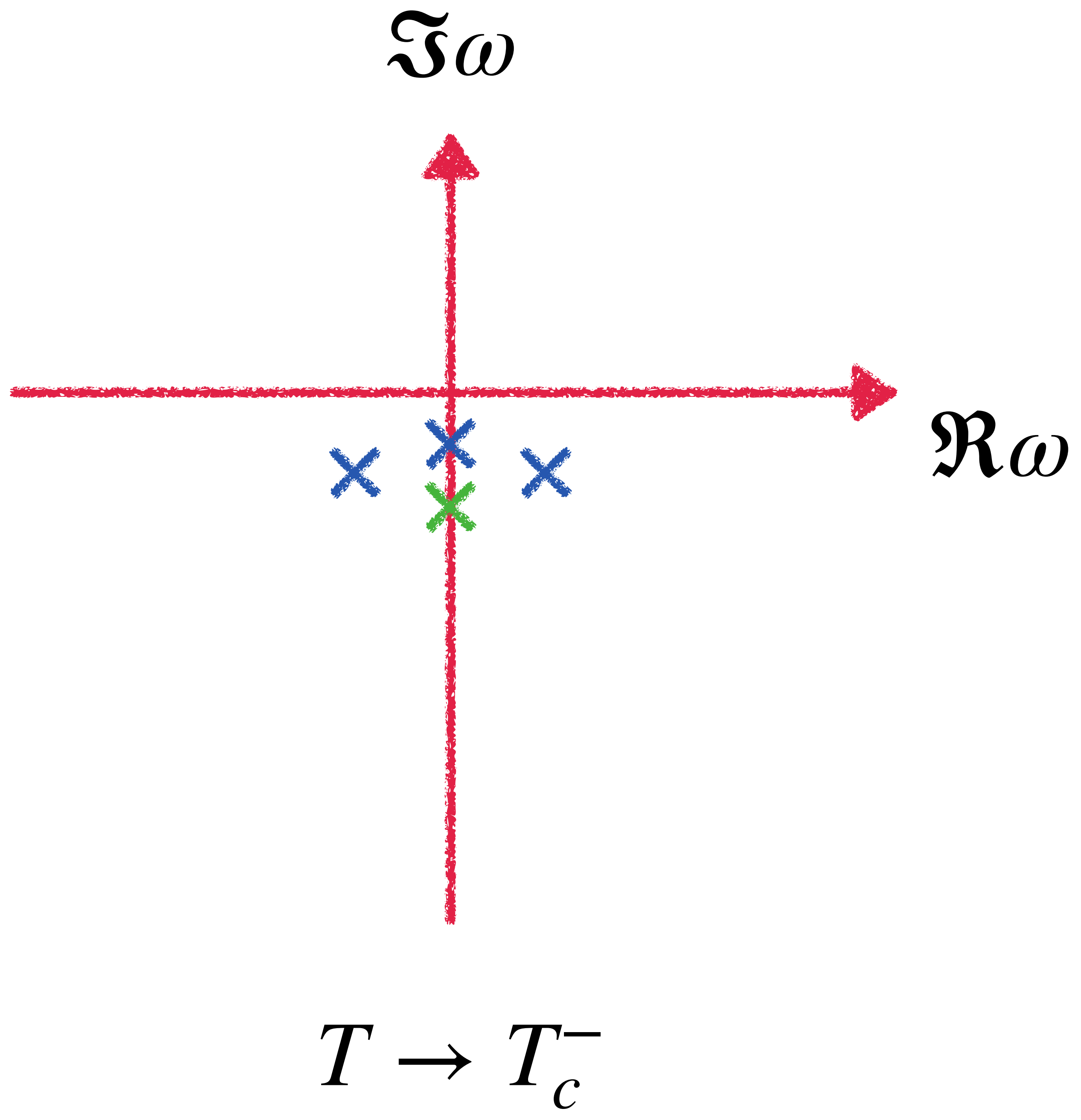
- Normal fluid parametrised by local temperature T and fluid velocity v^μ
- Superfluid parametrised by phase ϑ of vev $\langle \mathcal{O}_\psi \rangle = \langle \mathcal{O}_\psi \rangle_b \, e^{i\vartheta}$
- Express $T_{\mu\nu}$ and J_μ as functions of the fluctuations $T, v^\mu, v_s^\mu = \partial^\mu \vartheta$
- Solve the closed system

$$\nabla_\mu \langle T^{\mu\nu} \rangle = F^{\nu\mu} \langle J_\mu \rangle + \nabla^\nu \lambda \, \langle \mathcal{O}_\psi^* \rangle + \nabla^\nu \lambda^* \, \langle \mathcal{O}_\psi \rangle$$

$$\nabla_\alpha \langle J^\alpha \rangle = iq \left(\langle \mathcal{O}_\psi \rangle \lambda^* - \langle \mathcal{O}_\psi^* \rangle \lambda \right)$$

Hydrodynamics at $T < T_c$





- At $T \ll T_c$ linear response is dominated by hydro poles
- Higgs mode is integrated out
- As $T \to T_c^-$ Higgs pole becomes gapless $\to$ Transport coefficients blow up

▸ Include Higgs mode in hydro description

Generalities

- Isolate Higgs mode from rest of hydro fluctuations

- At infinite wavelength Higgs mode decouples from entropy and charge density

$$\partial_t \delta s = 0 \qquad\qquad \partial_t \delta \rho = 0$$

- Linear response of non-conserved scalar operators

Why holography

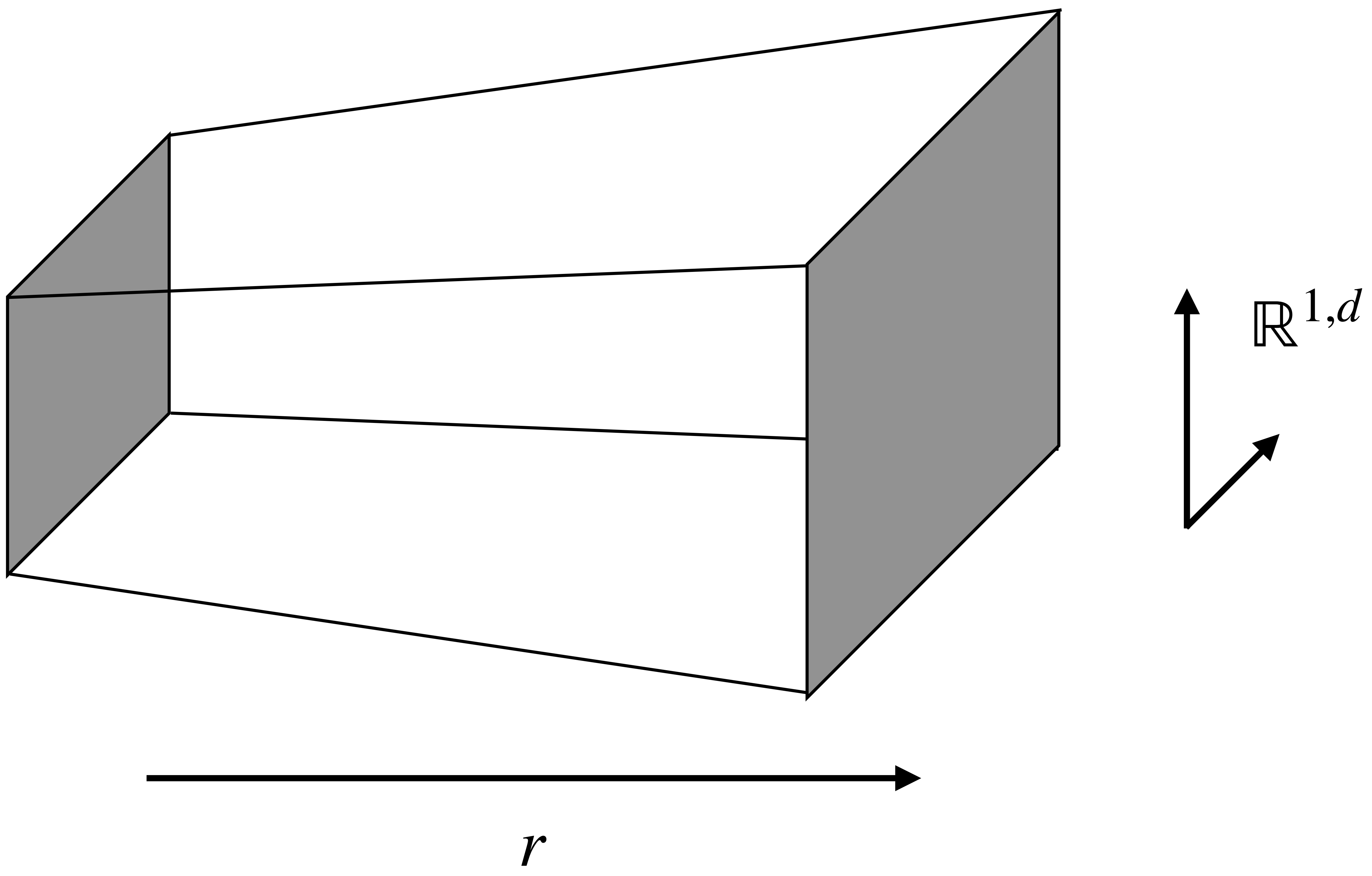
- Microscopic derivation $\Rightarrow$ Compute transport coefficients
- Valid away from $T_c \Rightarrow$ Ground states
- Real time dynamics

CFT Setup

Model superfluid transitions:

- CFT with a global $U(1)$ and charged operator $\mathcal{O}_\psi$
- Finite temperature T and chemical potential μ
- Deform by neutral relevant operator(s) $\mathcal{O}_\phi$ to introduce additional scale(s) $\phi_{(s)}$
- Phase transition at $T_c = T_c(\mu, \phi_{(s)})$

AdS/CFT



The vacuum of $CFT_{1,d}$ is modelled by AdS_{d+2}

$$ds^2 = r^2\left(-dt^2 + d\mathbf{x}_d^2\right) + \frac{dr^2}{r^2}$$

Holographic Setup

Boundary conditions of bulk fields correspond to sources in CFT:

- Metric ➜ Source for the stress tensor

$$ds^2 = r^2\left(-dt^2 + d\mathbf{x}_d^2 + \delta g_{\mu\nu}(\mathbf{x})dx^\mu dx^\nu\right) + \frac{dr^2}{r^2} + \cdots$$

- Gauge Field ➜ Source for $U(1)$ current

$$A = a_\mu(\mathbf{x})\, dx^\mu + \cdots$$

- Massive Scalar ➜ Source for boundary scalar with dimension Δ_ϕ

$$\phi(r, \mathbf{x}) = \frac{\phi_s(\mathbf{x})}{r^{d+1-\Delta_\phi}} + \cdots$$

Holographic Setup

- Boundary theory gets deformed to

$$S[\phi_s, a_\mu, \delta g_{\mu\nu}] = S_{CFT} + \int d^{d+1}x \left(\phi_s(x)\, \mathcal{O}(x) + a_\mu(x)\, J^\mu(x) + \frac{1}{2}\delta g_{\mu\nu}(x) T^{\mu\nu}(x) \right)$$

- Holographic conjecture relates partition functions

$$Z_{CFT}[\phi_s, a_\mu, \delta g_{\mu\nu}] = Z_{bulk}[\phi_s, a_\mu, \delta g_{\mu\nu}] \approx e^{iS_{bulk}[\phi_s, a_\mu, \delta g_{\mu\nu}]}$$

- Powerful tool to extract VEVs of operators

$$\langle \mathcal{O}(x) \rangle = \frac{1}{i} \frac{\delta}{\delta \phi_s(x)} \ln Z_{CFT}[\phi_s, a_\mu, \delta g_{\mu\nu}] \approx \frac{\delta}{\delta \phi_s(x)} S_{bulk}[\phi_s, a_\mu, \delta g_{\mu\nu}]$$

Setup

Minimal bulk action includes a complex scalar ψ and neutral scalar ϕ

$$\mathscr{L} = R - V(\phi, |\psi|^2) - \frac{1}{2}\partial_\mu\phi\, \partial^\mu\phi - \frac{1}{2}(D_\mu\psi)(D^\mu\psi)^* - \frac{1}{4}\tau(\phi, |\psi|^2)\, F^{\mu\nu}F_{\mu\nu}$$

$$D_\mu\psi = \nabla_\mu\psi + i\, q\, A_\mu\, \psi$$

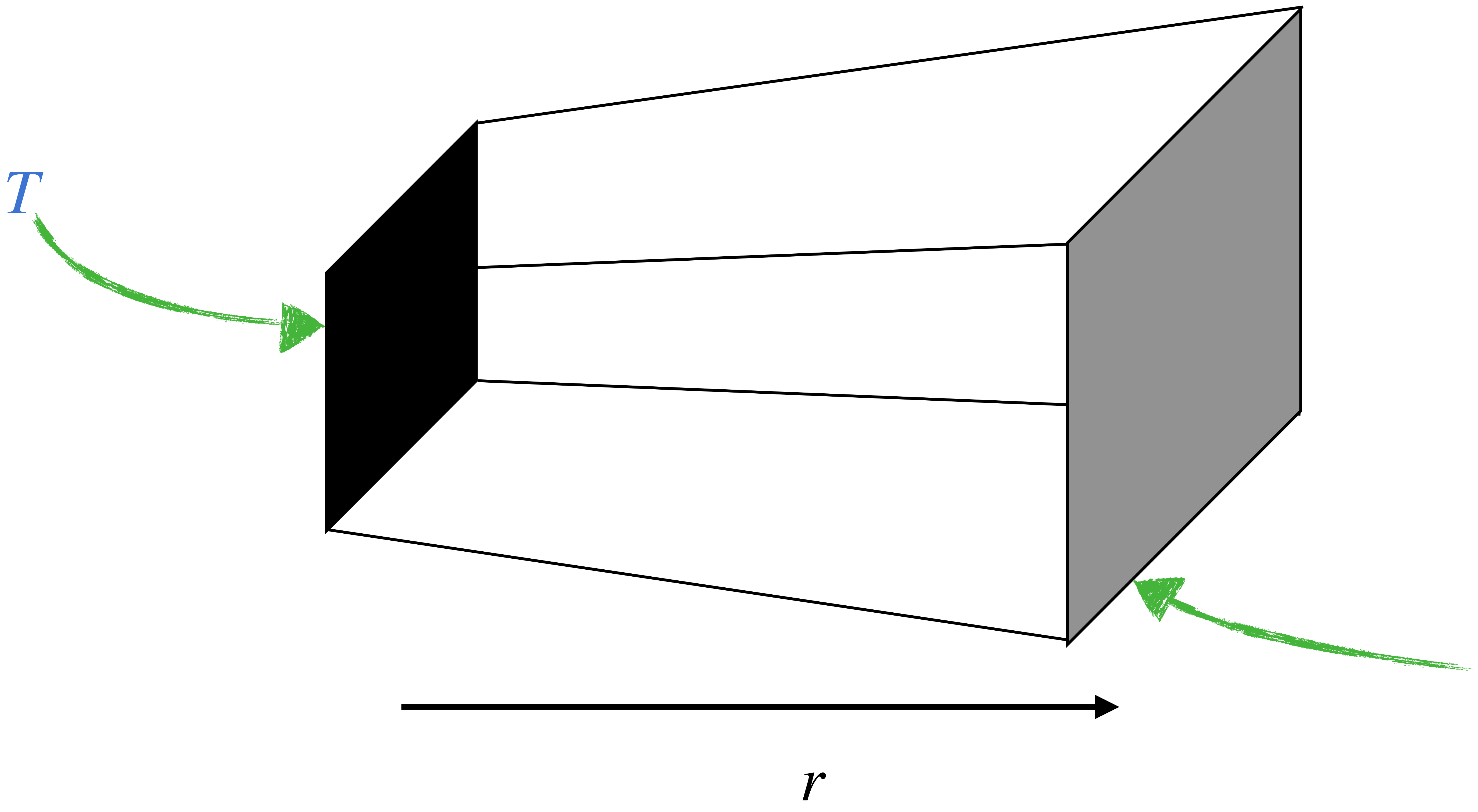
- Invariant under $\psi \to e^{-iq\Lambda}\psi,\ A_\mu \to A_\mu + \partial_\mu\Lambda$

- Scenario were we deform by neutral scalar ϕ and ψ breaks $U(1)$ below T_c

$$V \approx -6 + \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\psi^2|\psi|^2 + \cdots$$

- UV dimensions Δ_ϕ and Δ_ψ of dual operators fixed by m_ϕ^2 and m_ψ^2

Thermal States



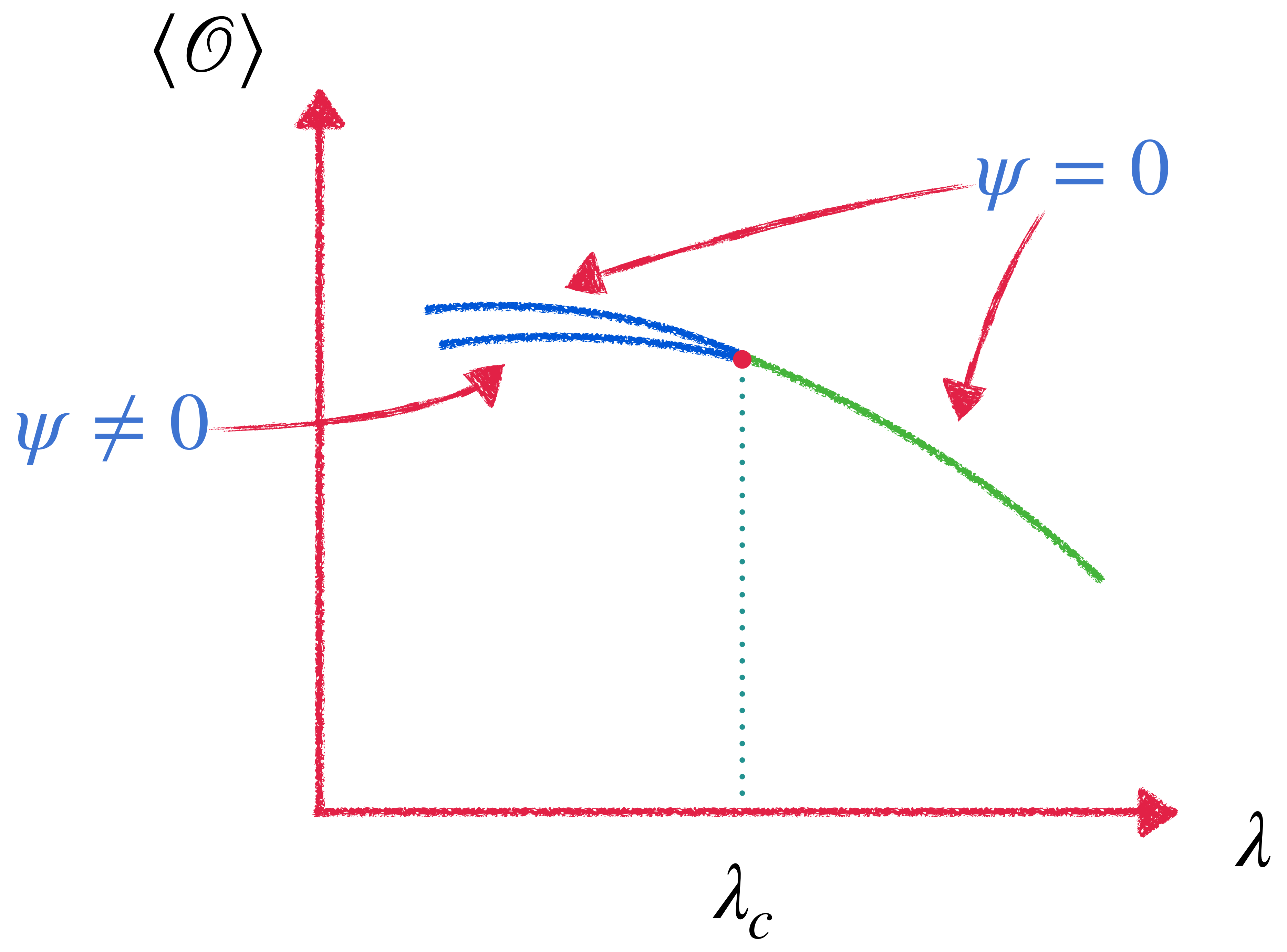
$$A_t \to \mu + \cdots$$

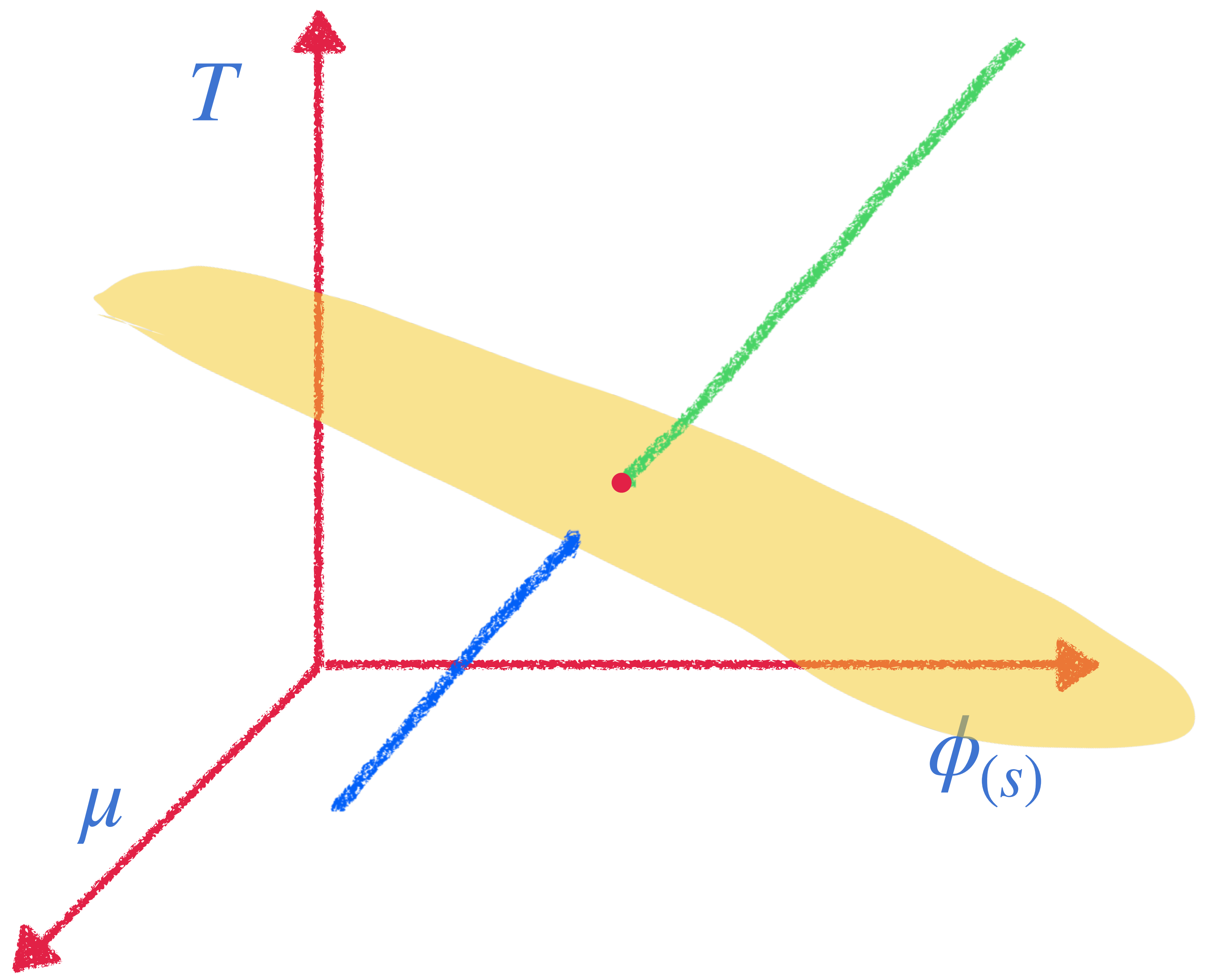
$$\phi(r) \to \frac{\phi_{(s)}}{r^{3-\Delta_\phi}} + \cdots + \frac{\phi_{(v)}}{r^{\Delta_\phi}} + \cdots$$

$$\psi(r) \to \frac{0}{r^{3-\Delta_\psi}} + \cdots + \frac{\psi_{(v)}}{r^{\Delta_\psi}} + \cdots$$

- Introduce planar event horizon at Hawking temperature T
- Fix μ and scalar sources $\phi_{(s)}$ on the boundary
- No source for complex scalar ψ

Phase Diagram

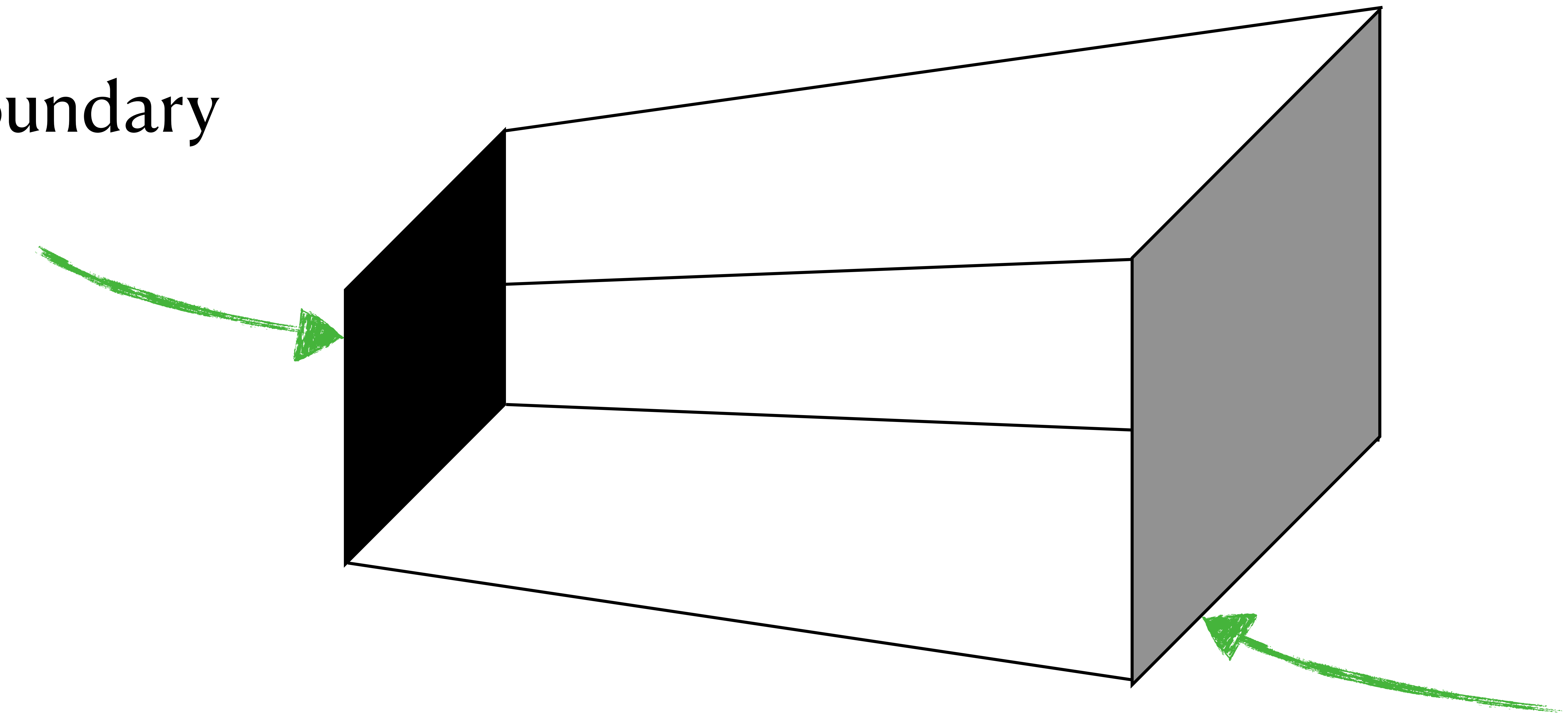




- Black holes parametrised by $(T, \mu, \phi_{(s)})$
- Consider curve $(T(\lambda), \mu(\lambda), \phi_{(s)}(\lambda))$
- Source free static mode $\delta\psi_c$ at $\lambda = \lambda_c$ Gubser Hartnoll, Herzog, Horowitz
- Follow instability for $\lambda < \lambda_c$ to construct broken phase black holes with $\psi \neq 0$

Linear Response

Infalling boundary conditions



$$\delta\phi(r) \to \frac{\delta\phi_{(s)}}{r^{3-\Delta_\phi}} + \cdots + \frac{\delta\phi_{(v)}}{r^{\Delta_\phi}} + \cdots$$

$$\delta\psi(r) \to \frac{\delta\psi_{(s)}}{r^{3-\Delta_\psi}} + \cdots + \frac{\delta\psi_{(v)}}{r^{\Delta_\psi}} + \cdots$$

- Introduce perturbative sources scalar $\delta\phi_{(s)}$ and $\delta\psi_{(s)}$
- Read off VEVs $\delta\langle\mathcal{O}_\phi\rangle$ and $\delta\langle\mathcal{O}_\psi\rangle$ from $\delta\phi_{(v)}$ and $\delta\psi_{(v)}$
- Extract retarded Green's functions
- Study source free Higgs mode

Higgs Mode Construction

- Construct source free Higgs mode
- At $\lambda = \lambda_c$ Higgs mode becomes the static mode $\delta\Phi_{*(0)} \to \delta\psi_c$
- Natural expansion parameter $\varepsilon = \lambda - \lambda_c$ with e.g. $\varepsilon^2 \propto 1 - T/T_c$

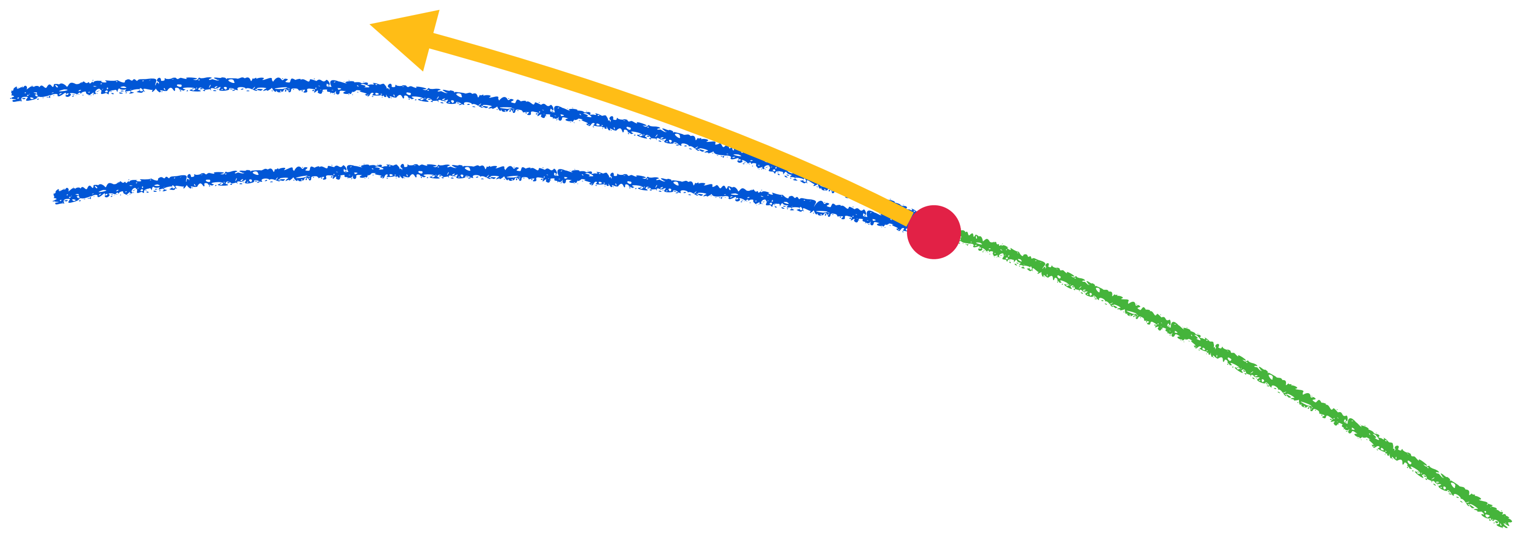
$$\delta\Phi = e^{-i\,\omega\,t}\,(\delta\Phi_{*(0)} + \varepsilon\,\delta\Phi_{(1)} + \cdots)$$

$$\omega = \varepsilon\,\omega_{[0]} + \varepsilon^2\,\omega_{[1]} + \cdots$$

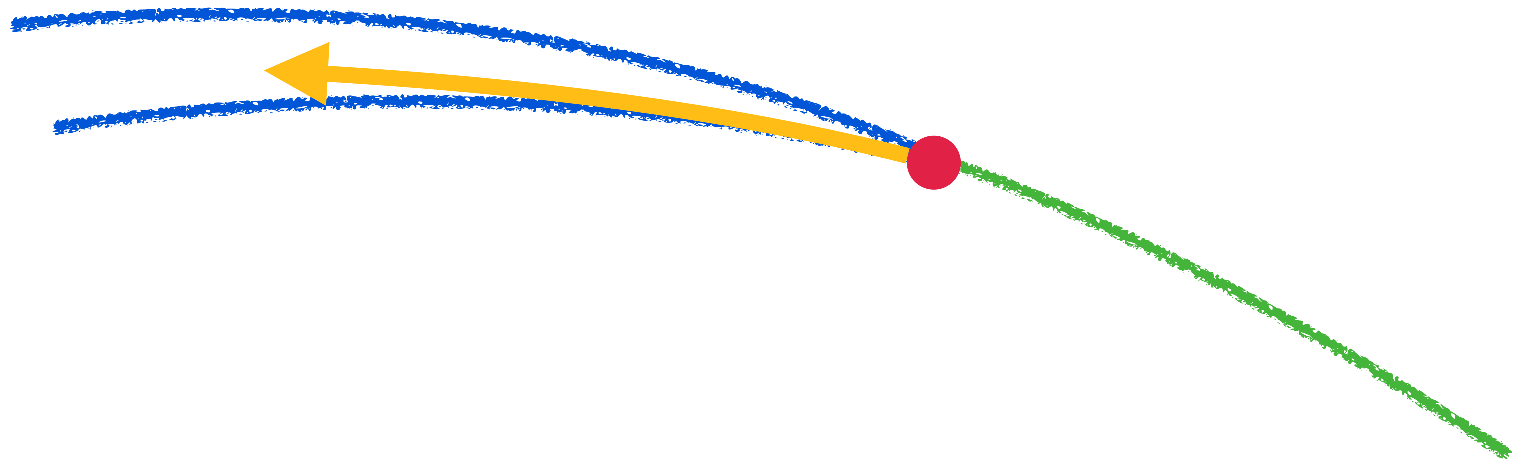
- Can show that $\omega_{[0]} = 0$
- Goal is to fix $\omega_{[1]}$

Higgs Mode Construction

- Expansion of branches around critical point



$$\Phi_\# = \Phi_c + \varepsilon^2\, \delta\Phi_{\#(1)} + \cdots$$



$$\Phi_* = \Phi_c + \varepsilon\, \delta\Phi_{*(0)} + \varepsilon^2\, \delta\Phi_{*(1)} + \cdots$$

- Equations of motion for Higgs mode solved by setting

$$\delta\Phi_{(1)} = \delta\Phi_{*(1)} - \delta\Phi_{\#(1)}$$

- Combination of variations so that $\delta s = 0, \delta\rho = 0$ and $\delta\phi_{(s)} = 0$

Higgs Mode

- Use holographic techniques to show Higgs mode decay rate is

Donos, Kailidis, Pantelidou

$$\omega_{dec} = -\frac{8\,\Delta E}{\varpi}$$

- Fixed by thermodynamics

$$\Delta E = E_{*}(s_c + \delta s, \rho_c + \delta\rho, \phi_{(s)c} + \delta\phi_{(s)}) - E_{\#}(s_c + \delta s, \rho_c + \delta\rho, \phi_{(s)c} + \delta\phi_{(s)})$$

- And dissipative coefficient fixed at the horizon

$$\varpi = \frac{s\,|\psi_h|^2}{4\pi} + \frac{16\,\pi}{s\,q^2\,|\psi_h|^2}\left(\rho_h - \rho\right)^2$$

Green's Functions

- Follow the same technique to introduce scalar sources $\delta\psi_{(s)} \propto \mathcal{O}(\varepsilon^2)$ and $\delta\phi_{(s)} \propto \mathcal{O}(\varepsilon)$
- Read off VEVs to find

$$G_{\mathcal{O}_\rho \mathcal{O}_\rho}(\omega) = \frac{(\Delta\langle\mathcal{O}_\psi\rangle)^2}{\varpi\left(-i\,\omega + \omega_{gap}\right)} \qquad G_{\mathcal{O}_\phi \mathcal{O}_\rho}(\omega) = 2\,\frac{\Delta\langle\mathcal{O}_\psi\rangle\,\Delta\langle\mathcal{O}_\phi\rangle}{\varpi\left(-i\,\omega + \omega_{gap}\right)}$$

$$G_{\mathcal{O}_\phi \mathcal{O}_\phi}(\omega) = 4\,\frac{\left(\Delta\langle\mathcal{O}_\phi\rangle\right)^2}{\varpi\left(-i\,\omega + \omega_{gap}\right)} + \partial_{\phi_{(s)}}\langle\mathcal{O}_\phi\rangle_{\#}\Big|_{s,\varrho}$$

- With $\Delta\langle\mathcal{O}\rangle = \langle\mathcal{O}\rangle_*(s_c + \delta s, \rho_c + \delta\rho, \phi_{(s)c} + \delta\phi_{(s)}) - \langle\mathcal{O}\rangle_\#(s_c + \delta s, \rho_c + \delta\rho, \phi_{(s)c} + \delta\phi_{(s)})$
- Reminiscent of memory matrix formalism with fixed susceptibilities and gap

Conclusions/Outlook

- Holography as a tool to study universal behaviour
- Connect with standard field theory approaches

  Halperin, Hohenberg   Glorioso, Liu
- Complete set of Green's functions
- Amplitude modes for broken spacetime symmetries

  Donos, Gauntlett, Pantelidou
- Enlarge hydro of broken phase to include amplitude mode