Thermo-electric transport properties in holographic models with pinned charge density waves

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Strange metals: from the Hubbard model to AdS/CFT

Institute of Physics Belgrade (IPB), online, 25 May'22

References

Based on:

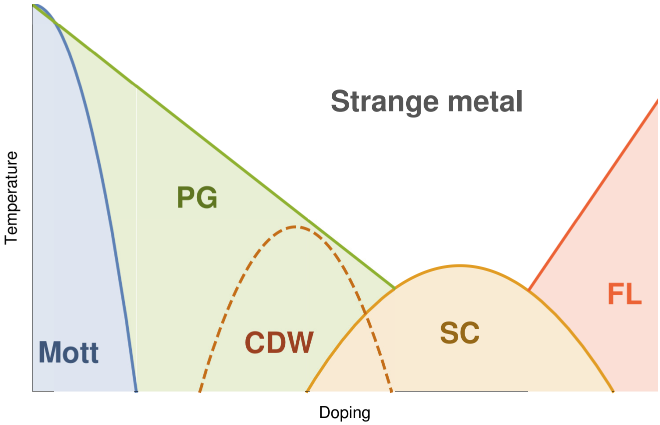
Tomas Andrade, A.K.,
arXiv:2203.10038

Outline

1. **Effective transport in CDW**
2. **Holographic model with pinned CDW**
3. **DC Transport in strong order regime**
4. **Conclusion**

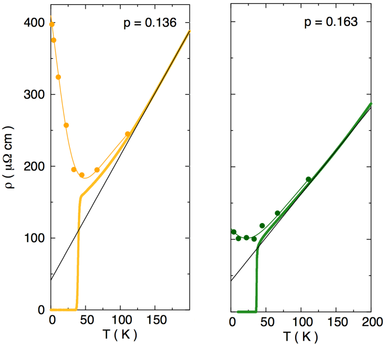
CDW in cuprates

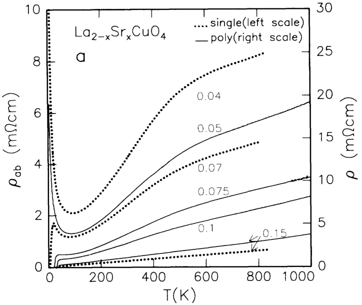
CDW is a clear part of the cuprate HTSC phase diagram



Transport in CDW

It is characterized by some peculiar transport features:
The smooth upturn of the resistivity

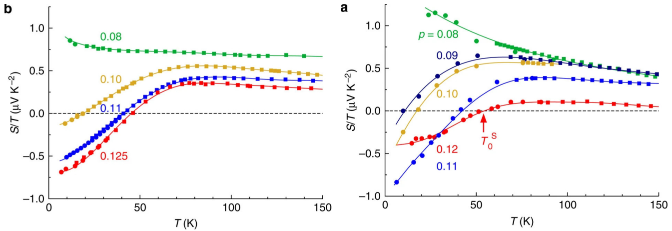




Teillefer et al, arXiv:1606.04491 H.Takagi *et al.* PRL 69, 2975

Transport in CDW

It is characterized by some peculiar transport features:
Change of sign in the thermopower



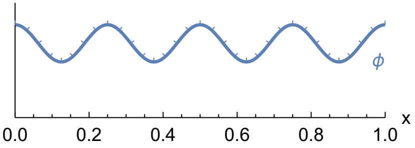
a)EuBCO, b)YBCO:
Taillefer et al. arXiv:1102.0984 , Nat Commun 2,432 (2011)

Effective theory of transport in CDW

Effective theory of pinned CDW

Let's consider the EFT of a Goldstone mode of the broken translations:

$$\phi(x) - \text{a point of CDW located at position } x$$
$$\phi(x) \to \phi(x) + \delta\phi - \text{a shift of CDW, or redefinition of internal ruler}$$
$$\partial_t \phi(x) - \text{velocity of CDW at position } x$$



J.Armas, A.Jain 2001.07357, recall also talk by B.Gouteraux

Effective theory of pinned CDW

Conservation laws include pinning term Γ and Goldstone mass m_ϕ^2

$$
\begin{aligned}
\nabla_\mu T^\mu_t &= 0, \\
\nabla_\mu J^\mu &= 0 \\
\nabla_\mu T^\mu_x &= -\rho \partial_t \delta A_x + \Gamma T_{tx} - G m_\phi^2 \delta\phi
\end{aligned}
$$

L. V. Delacrétaz, et al, 1702.05104

The theory can be formulated having arbitrary metric in mind.

$$\text{crystal metric: } h^{IJ} = g^{\mu\nu} e^I_\mu e^J_\nu, \qquad e^I_\mu \equiv \partial_\mu \phi^I$$

$$\text{strain tensor: } u_{IJ} = \frac{1}{2}(H_{IJ} - h_{IJ})$$

Effective theory of pinned CDW

Constitutive relations involve γ – a contribution of moving CDW to the current. In conventional gapped CDW: $\sigma_q = 0$, $\xi = 0$

$$J^x = \rho\delta u_x + \gamma(\partial_t\delta\phi - \delta u_x) - \sigma_q\left(T_0\partial_x\frac{\mu}{T} + \partial_t\delta A_x\right),$$

$$T^{xx} = P + \Big(s\delta T + \rho\delta\mu + (\zeta+\eta)\partial_x\delta u_x - (B+G)\partial_x\delta\phi - 2\eta\partial_x\delta g_{tx}\Big).$$

A decay rate Ω must be added to the Josephson relation:

$$\partial_t\delta\phi = \delta u_x + \xi(B+G)\partial_x^2\delta\phi - \xi\gamma'\left(T_0\partial_x\frac{\mu}{T} + \partial_t\delta A_x\right) - \Omega\delta\phi.$$

Consistency and locality requires

$$\gamma' = -\gamma, \qquad \Omega = \xi m_\phi^2 G.$$

J.Armas, A.Jain 2001.07357,
L.V. Delacrétaz, B. Goutéraux, V. Ziogas 2111.13459

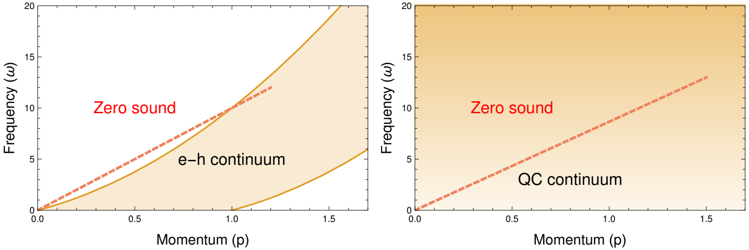
also talk by B. Groutéreaux

Decay of the Goldstone

Note that ξ measures the diffusivity of the Goldstone

$$\partial_t \delta\phi = \delta u_x + \xi (B + G) \partial_x^2 \delta\phi - \xi \gamma' \left(T_0 \partial_x \frac{\mu}{T} + \partial_t \delta A_x \right) - \Omega \delta\phi.$$

$$\omega(k) = -i\Omega - i\xi(B + G)k^2$$



A. Romero-Bermúdez et al 1812.03968

also Jan's talk

Thermo-electric transport

Let us look at full matrix of thermo-electric conductivities

$$\begin{pmatrix} J^x \\ Q^x \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\bar{\alpha} & T\bar{\kappa} \end{pmatrix} \begin{pmatrix} E_x \\ -\frac{\partial_x T}{T} \end{pmatrix}$$

$$Q^x = T^{xt} - \mu_0 J^x$$

$$\sigma = \langle J^x J^x \rangle, \qquad T\alpha = \langle J^x Q^x \rangle, \qquad T\bar{\kappa} = \langle Q^x Q^x \rangle$$

We can evaluate it by taking the variations with respect to the perturbative sources $\delta A_x(\omega)$ and $\delta g_{tx}(\omega)$

Effective theory of pinned CDW

The AC conductivities can be evaluated

$$\sigma(\omega) = \tilde{\sigma} + \frac{\tilde{\rho}^2(\Omega - i\omega) - \tilde{\gamma}^2\omega_0^2(\Gamma - i\omega) - 2\tilde{\rho}\tilde{\gamma}\omega_0^2}{\mu_0^2\chi_{\pi\pi}\left((\Gamma - i\omega)(\Omega - i\omega) + \omega_0^2\right)}$$

$$\frac{T}{\mu_0}\alpha(\omega) = -\tilde{\sigma} + \frac{\tilde{\rho}\tilde{s}\left((\Omega - i\omega) + \tilde{\gamma}^2\omega_0^2(\Gamma - i\omega) - (\tilde{s} - \tilde{\rho})\tilde{\gamma}\omega_0^2\right.}{\mu_0^2\chi_{\pi\pi}\left((\Gamma - i\omega)(\Omega - i\omega) + \omega_0^2\right)}$$

$$\frac{T}{\mu_0^2}\bar{\kappa}(\omega) = \tilde{\sigma} + \frac{\tilde{s}^2(\Omega - i\omega) - \tilde{\gamma}^2\omega_0^2(\Gamma - i\omega) + 2\tilde{s}\tilde{\gamma}\omega_0^2}{\mu_0^2\chi_{\pi\pi}\left((\Gamma - i\omega)(\Omega - i\omega) + \omega_0^2\right)},$$

where $\tilde{\rho} = \mu_0\rho$, $\tilde{s} = T_0 s$, $\chi_{\pi\pi} = \tilde{\rho} + \tilde{s}$ and

$$\tilde{\sigma} = \sigma_q + \xi\gamma^2, \quad \omega_0^2 = \frac{G m_\phi^2}{\chi_{\pi\pi}}, \quad \tilde{\gamma} = \mu_0\chi_{\pi\pi}, \quad \Omega = \xi m_\phi^2 G$$

At $\omega_0 > \Omega$ these are the asymmetric peaks at $\omega \approx \pm\omega_0 - i(\Gamma + \Omega)$

Effective theory of pinned CDW

DC conductivities have simple form

$$\sigma = \sigma_q + \xi \frac{(\rho - \gamma)^2}{1 + \xi \Gamma \chi_{\pi\pi}},$$

$$\frac{T}{\mu_0}\alpha = -\sigma_q + \xi \frac{(\rho - \gamma)(\frac{sT}{\mu_0} + \gamma)}{1 + \xi \Gamma \chi_{\pi\pi}}$$

$$\frac{T}{\mu_0^2}\bar{\kappa} = \sigma_q + \xi \frac{(\frac{sT}{\mu_0} + \gamma)^2}{1 + \xi \Gamma \chi_{\pi\pi}}$$

At small pinning scale Γ, DC transport is insensitive to impurities!

In conventional gapped CDW in Fermi liquid: $\sigma_q = 0$, $\xi = 0$, therefore **conventional pinned CDW is an insulator.**

Holographic model with pinned CDW

Holographic model with pinned CDW

We consider a holographic model which develops spontaneous CDW order

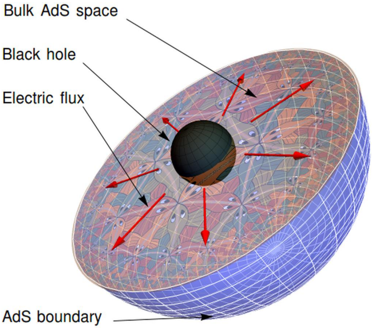
$$S = \int d^4x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}(\partial\psi)^2 - \frac{\tau(\psi)}{4}F^2 - W(\psi)\right) - \frac{1}{2}\int \vartheta(\psi)F\wedge F.$$

With potentials

$$\tau(\psi) \approx 1 + \dots,$$
$$W(\psi) \approx -\psi^2 + \dots,$$
$$\vartheta(\psi) \approx \frac{c_1}{2\sqrt{6}}\psi + \dots$$

And explicit ionic lattice

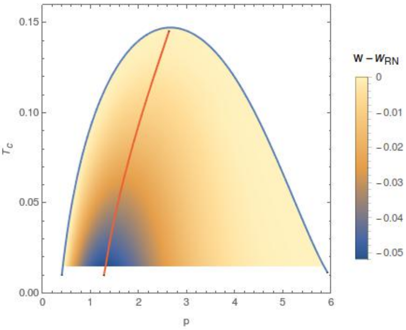
$$\mu = \mu_0(1 + A\cos(px))$$

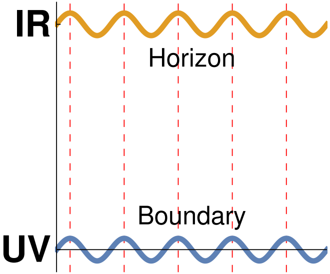


Holographic model with pinned CDW

The spontaneous structure arises as an instability of horizon

$$\delta A_y \sim \sin(kx), \qquad \delta\psi \sim \cos(kx)$$



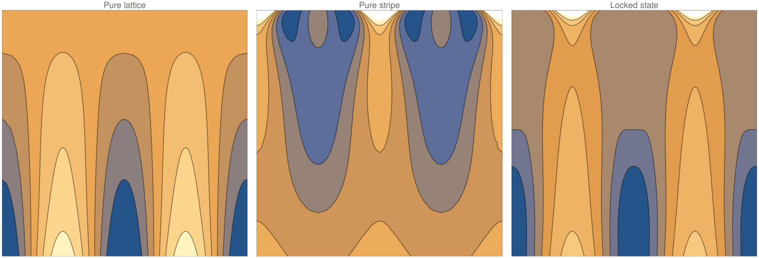


A.Donos, G. Gauntlett 1106.2004; A. Donos 1303.7211

G.T. Horowitz, J. E. Santos, D. Tong 1209.1098

Holographic model with pinned CDW

The spontaneous structure arises as an instability of horizon

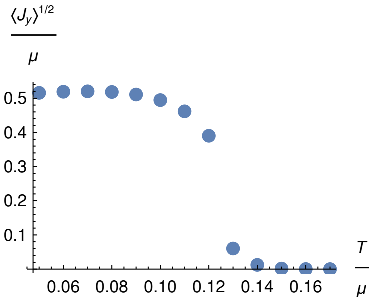


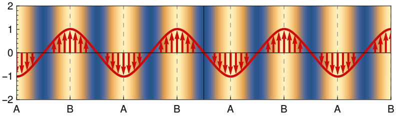
T.Andrade, A.K. et al, 1710.05791, Nat.phys.,14.10(2018):1049

Holographic model with pinned CDW

The order parameter grows as the temperature is lowered

$$\delta A_y \sim \sin(kx), \qquad \delta\psi \sim \cos(kx)$$
$$\delta\rho \sim \rho^{(0)} + \rho^{(2)} \sin(2kx)$$





Transport properties

Thermo-electric transport

Again, the matrix of thermo-electric conductivities can be computed by turning on perturbative sources $\delta A_x(\omega)$ and $\delta g_{tx}(\omega)$

$$\begin{pmatrix} J^x \\ Q^x \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\bar{\alpha} & T\bar{\kappa} \end{pmatrix} \begin{pmatrix} E_x \\ -\frac{\partial_x T}{T} \end{pmatrix}$$

$$Q^x = T^{xt} - \mu_0 J^x$$

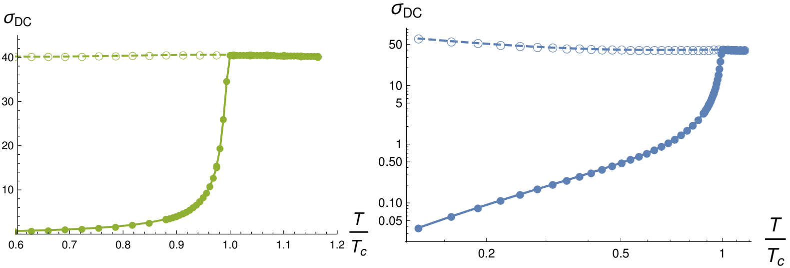
$$\sigma = \langle J^x J^x \rangle, \qquad T\alpha = \langle J^x Q^x \rangle, \qquad T\bar{\kappa} = \langle Q^x Q^x \rangle$$

i.e.

$$\langle T^{xt} T^{xt} \rangle = \frac{\delta^2 S_{\mathrm{AdS}}}{\delta g_{tx} \delta g_{tx}}$$

Electric DC conductivity

The transport is greatly affected by the emergence of order parameter

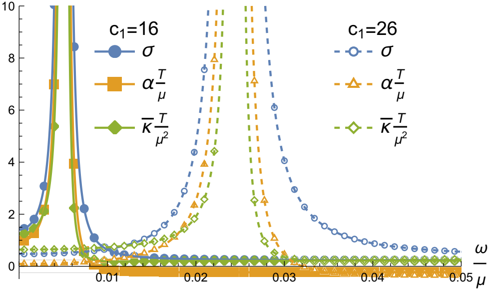


The conductivity drops, but it is not gapped ($\sim e^{-\frac{\Delta}{T}}$)like it should be in a pinned CDW. Our aim is to characterize this remaining transport at $T \ll T_c$

T.Andrade, A.K. et al, 1710.05791, Nat.phys.,14.10(2018):1049

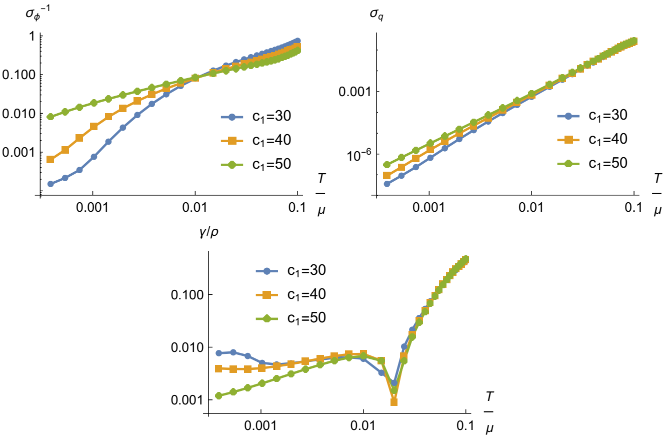
AC conductivities

The AC conductivities display the expected peaks, therefore **EFT is applicable**



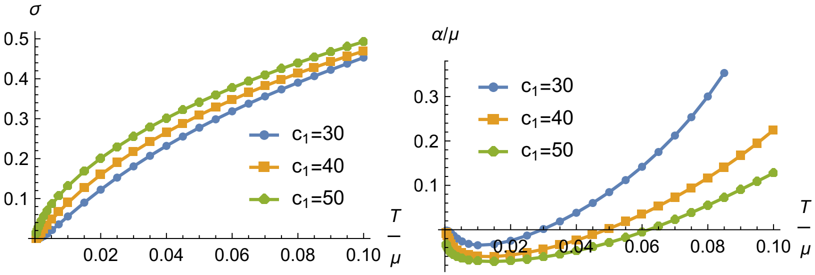
EFT parameters

The EFT parameters display model dependent power-laws in T



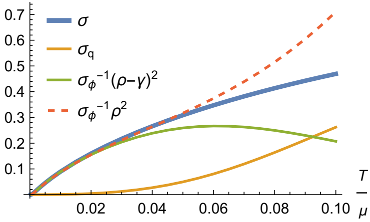
DC conductivities

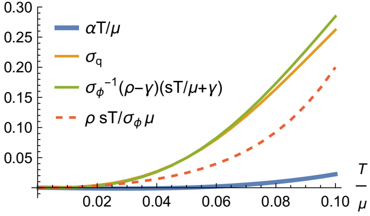
The DC conductivities have the similar unconventional features as in cuprates

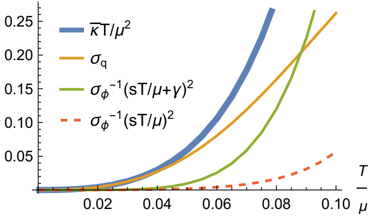


$$\sigma = \sigma_q + \xi \frac{(\rho-\gamma)^2}{1+\xi\Gamma\chi_{\pi\pi}}, \qquad \frac{T}{\mu_0}\alpha = -\sigma_q + \xi \frac{(\rho-\gamma)(\frac{sT}{\mu_0}+\gamma)}{1+\xi\Gamma\chi_{\pi\pi}}$$

Contributions







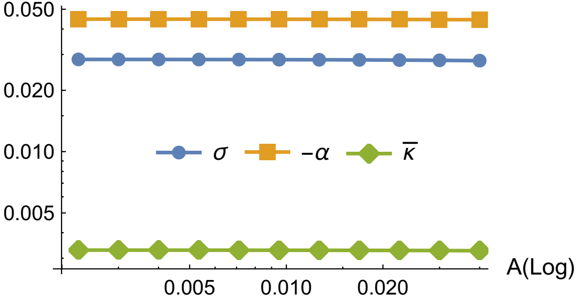
$$\sigma = \sigma_q + \xi \frac{(\rho - \gamma)^2}{1 + \xi \Gamma \chi_{\pi\pi}}$$

$$\frac{T}{\mu_0}\alpha = -\sigma_q + \xi \frac{(\rho - \gamma)(\frac{sT}{\mu_0} + \gamma)}{1 + \xi \Gamma \chi_{\pi\pi}}$$

$$\frac{T}{\mu_0^2}\bar{\kappa} = \sigma_q + \xi \frac{(\frac{sT}{\mu_0} + \gamma)^2}{1 + \xi \Gamma \chi_{\pi\pi}}$$

The role of impurities

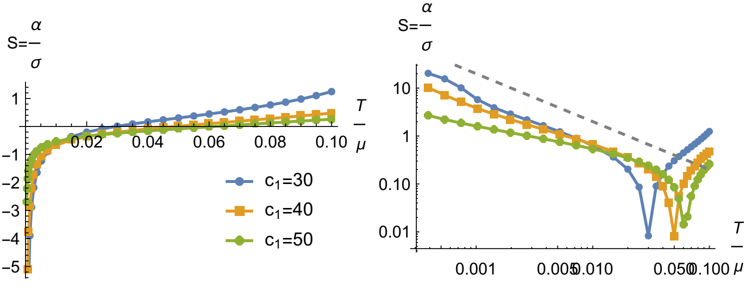
The DC transport is not controlled by the scale of pinning



$$\sigma = \sigma_q + \xi \frac{(\rho - \gamma)^2}{1 + \xi \Gamma \chi_{\pi\pi}}, \qquad \frac{T}{\mu_0}\alpha = -\sigma_q + \xi \frac{(\rho - \gamma)(\frac{sT}{\mu_0} + \gamma)}{1 + \xi \Gamma \chi_{\pi\pi}}$$

Seebeck coefficient

Seebeck coefficient in this particular model diverges. It's behavior is controlled by the IR scaling exponents



c.f. the talk by Antoine Georges

Conclusion

- Effective theory of transport in pinned CDW includes several parameters, which are usually set to zero in conventional treatments
- Holography provides an example of the system, where these parameters play a role
- The expanded phenomenology displays gapless insulators and change of sign in thermo-power, due to a balance between contributions
- The EFT framework is useful for the analysis of experimental data